The basis of Distributional Learning for the inference of Non-Regular Languages

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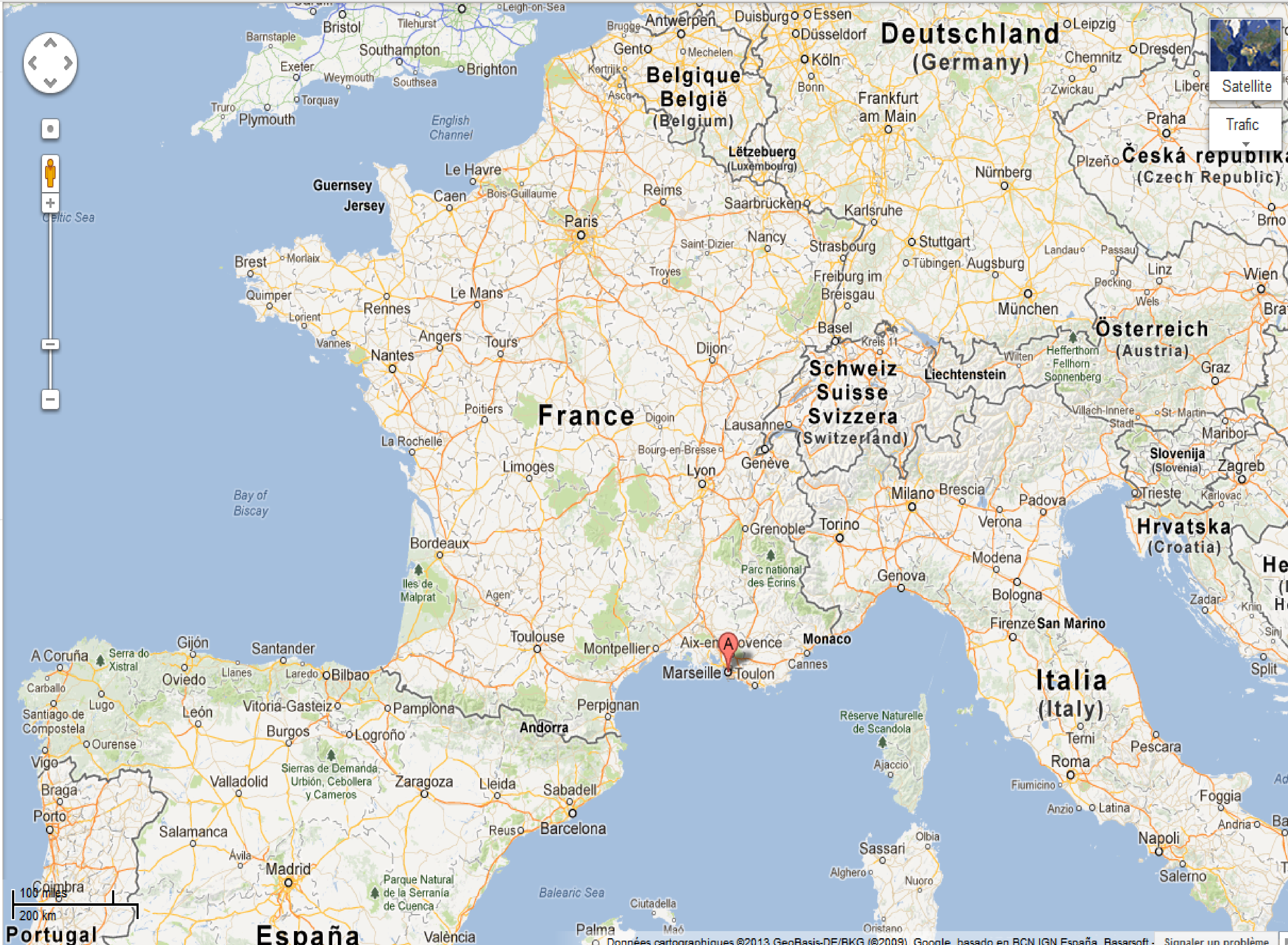
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Outline

- **Introduction**
- Learning Substitutable languages
- Prime congruence classes
- Extension to tree and graph grammars
- A dual approach
- Conclusion

Some basic definitions (Formal Language Theory)

- An alphabet is a finite set of symbols. Ex.:
  - $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
  - $\Sigma = \{A, C, T, G\}$
  - Σ = set of Part-Of-Speech tags
  - $\Sigma = \{0, 1\}$, usually denoted $\{a, b\}$
- A word/string on an alphabet is a finite sequence of symbols
  - 000042 or 84 or ε
  - VBN-TL NNS-TL IN-TL NP-TL

Some basic definitions (Formal Language Theory)

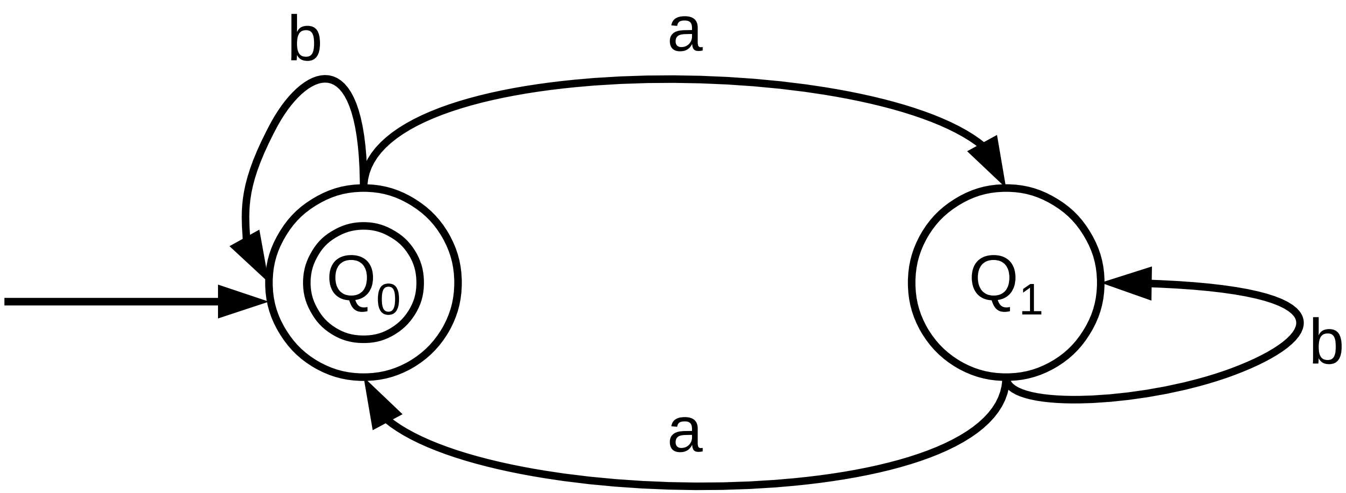
- A language over an alphabet is a (possibly non-finite) set of strings over this alphabet.
  - L = {10, 11, 12, 13, 14, 15, 16, 17, 18, 19}
  - L = all natural numbers
  - L = all DNA sequences that correspond to a gene
  - L = all POS tag sequences that correspond to an English sentence
  - $L = \{a^n b^n : n>0\}$

Some basic definitions (Formal Language Theory)

- If a language is infinite (or too big to be finitely represented), we need a finite representation to be able to handle it : we call this representation a **grammar**.
- Well-known grammar classes : the Chomsky hierrachy
  - Regular grammars: productions $A \rightarrow aB$ or $A \rightarrow \varepsilon$
  - Context-free grammars: $A \rightarrow BC$ or $A \rightarrow a$ (normal form)
  - Context sensitive grammars: $\alpha A \beta \rightarrow \alpha \gamma \beta$
- With each of these classes of grammars is associated a class of languages.

Some basic definitions (Formal Language Theory)

- DFA:



- Regular grammars:

  $Q_0 \rightarrow b\ Q_0,\ Q_0 \rightarrow a\ Q_1,\ Q_0 \rightarrow \varepsilon$

  $Q_1 \rightarrow b\ Q_1,\ Q_1 \rightarrow a\ Q_0$

Some basic definitions (Formal Language Theory)

- Context-free grammars:
  - The alphabet of the language : Σ
  - A finite set of variables (called non-terminals) : N
  - A set of context-free rules: $P \subset N \rightarrow (N \cup \Sigma)^*$
  - A special non-terminal (the axiom) : S
- The language represented by a context-free grammar is the set of strings over Σ that one can obtain from the axiom using the rules of P.

Some basic definitions (Formal Language Theory)

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- Example: $\Sigma=\{a, b\}$, $N=\{S\}$, $P=\{S \rightarrow a\ S\ b,\ S \rightarrow a\ b\ \}$

What is the language of this grammar?

Some basic definitions (Formal Language Theory)

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- Example: $\Sigma=\{a, b\}$, $N=\{S\}$, $P=\{S \rightarrow a\ S\ b,\ S \rightarrow a\ b\ \}$

  What is the language of this grammar?

  - $\{a^n b^n : n>0\}$
  - $n = 3$: $S \rightarrow a\ S\ b \rightarrow a\ a\ S\ b\ b \rightarrow a\ a\ a\ b\ b\ b$

    We write $S \rightarrow^* aaabbb$

Grammatical Inference

- We are interested in algorithms that are able to learn a class of languages using a given class of grammars:
  - A learning algorithm is fed with data corresponding to an unknown language of the class,
  - It outputs a hypothesis (i.e. a grammar of the class) that is a representation of a language,
  - If the outputed grammar is an “acceptable” representation of the unknown language then the algorithm has learnt the language
  - If it can learn all languages of the class, then we say that the algorithm learns the class.

Grammatical Inference

- When can we say that an algorithm is able to learn?
  - Experimentally validate (e.g. Cross-validation)

Corpus		
a a a a a b b b b b	Yes	Learning sample
b a b b b a a b b	No	
a b b	No	
a a a a a a a a a a a b b b b b b b b b b b	Yes	
a b a b a b a b	No	
a b	Yes	test sample

  - Fulfills conditions of a formal definition of learning

Grammatical Inference

- When can we say that an algorithm is able to learn?
  - Practically validate (e.g. cross-validation)
  - Fulfills conditions of a formal definition of learning:
    - Probably Approximatively Correct (PAC) paradigm

The probability that the error rate of the output *h* of the agorithm is greater than a epsilon is less than a delta: $\Pr(\mathrm{error}(h) < \varepsilon) > 1 - \delta$

Grammatical Inference

- When can we say that an algorithm is able to learn?
  - Practically validate (e.g. cross-validation)
  - Fulfills conditions of a formal definition of learning:
    - Probably Approximatively Correct (PAC) paradigm
    - Identification in the limit

Input: an infinite (complete) sequence of data

Behaviour: for each new data a hypothesis is outputed

Learning: for each possible sequence, there exists a moment at which the algorithm **converges** to a hypothesis that is equivalent to the **target** grammar, and it never changes its hypothesis after.

Grammatical Inference

- What kind of data?
  - Examples (of sentences, DNA codes, bird songs, ...)
  - Example and counter-examples
  - Structured examples (trees, skeletons, ...)
  - Queries to an oracle
  - ...

Grammatical Inference

- Nice results for regular languages:
  - Efficient identification from positive and negative examples, RPNI [Oncina & Garcia, 92]
  - Identification from positive examples only of subclasses: reversible [Angluin, 82], locally testable, ...
  - PAC-learning of the whole class from positive examples (with restrictions on the distribution of examples) [Clark & Thollard, 02]
  - Learning of the whole class using membership and equivalence queries [Angluin, 87]

Grammatical Inference

- Nice results for regular grammars:
- Few postives results for context-free grammars (prior to the works presented today)
  - Efficient identification of small subclasses from postive and negative examples (reduction to the regular case)
  - Identification of a very restrictive subclass from positive examples (very simple grammars [Yolomori, 02])
  - The whole class from skeletons [Sakakibara, 92]
- No positive result for contex-sensitive grammars

Grammatical Inference

- Why are regular languages a success story?
  - Strong link between the representation and the structure of the language (residual, Nerode equivalence classes, ...).
- Slogan: “The structure of the representation should be based on the structure of the language, not something arbitrarily imposed on it from outside”
  - Identify some structure in the language
  - Show how that structure can be observed
  - Construct a representation based on that structure

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Key idea

- Learn a structurally defined class of languages
- Use only examples of the language
- Rely on the notion of *Context:*

A string u appear in the context (l,r) in a string w if w=lur.

The set of all contexts of u in a language L is written $C_L(u) = \{ (l,r) : lur \text{ in } L\}$.

For instance, the substring ab appears in the context (a,b) in the word aabb.

Syntactic congruence

- Well-studied relation structuring languages
- *u* and *v* are syntactically congruent w.r.t. a language L iff for all *l*,*r* in Σ*, *lur* is in L iff *lvr* is in L ($u \equiv_L v$).
- In term of context, $u \equiv_L v$ iff $C_L(u) = C_L(v)$.
- [u] is congruence class of *u*, i.e. the set of all substrings *v* such that $u \equiv_L v$
- Example: L={bcb,baab,cb,aab}, $c \equiv_L aa$, $bcb \equiv_L baab$, [bcb] = [baab] ≠ [cb]

A weaker relation

- *The weak substitutability*:

  u and *v* are weakly substitutable w.r.t. a language L, iff there exist *l*,*r* in Σ^*, *lur* is in L iff *lvr* is in L

  Notation : $u \approx_L v$

- In term of set of contexts, $u \approx_L v$ if and only if $C_L(u) \cap C_L(v) \neq \emptyset$

Substitutability

The syntactic congruence is the most interesting: u and v always appear in the same context (then they can be generated by the same non-terminal, for instance).

But: from a finite set of examples, we can only observe the weak substitutability.

We can unify these notions in order to ensure the observability of the syntactic congruence

Substitutable languages

- A language L is substitutable iff for all u and v in Σ^*, $u \approx_L v$ implies $u \equiv_L v$, i.e. the weak substitutability implies the syntactic congruence.

- The sets of contexts of two substrings of words of L are either disjoint or identical.
- In other words:

  If lur, lvr and l'ur' are in L then l'vr' is in L.

Examples

- Σ^* is substitutable.
- $\{a^n \mid n>0\}$ is substitutable (all contexts of a substring have the form (a^k,a^l)).
- $\{wcw^R \mid w \text{ in } (a,b)^*\}$ is substitutable.
- $\{a^n cb^n \mid n>0\}$ is substitutable.
- $\{w : |w|_a = |w|_b \text{ and } |w|_c = |w|_d\}$ is substitutable.

- $\{a^n b^n \mid n>0\}$ is not substitutable: for instance, we have $a \approx_L aab$ but not $a \equiv_L aab$
- $\{a,aa\}$ is not substitutable (a and aa share the context $(\varepsilon,\varepsilon)$ but not the context (ε,a))

Algorithm: main ideas

- We want to compute the syntactic classes of the language from the examples, together with their mutual structure.
- We are going to use the fact that $[u][v] \subseteq [uv]$ by creating the rules $[uv] \rightarrow [u][v]$

- Algorithmic trick: the *Substitution graph*
  - each distinct substring of the learning sample is a node.
  - There is an edge between two nodes if they appear in the same context(s).

Running Example

LS={c;aca;bcb;abcba;aacaa} (palindrome with a center marked)

c bcba abcb

abcba aca acaa ac

bcb ca aaca

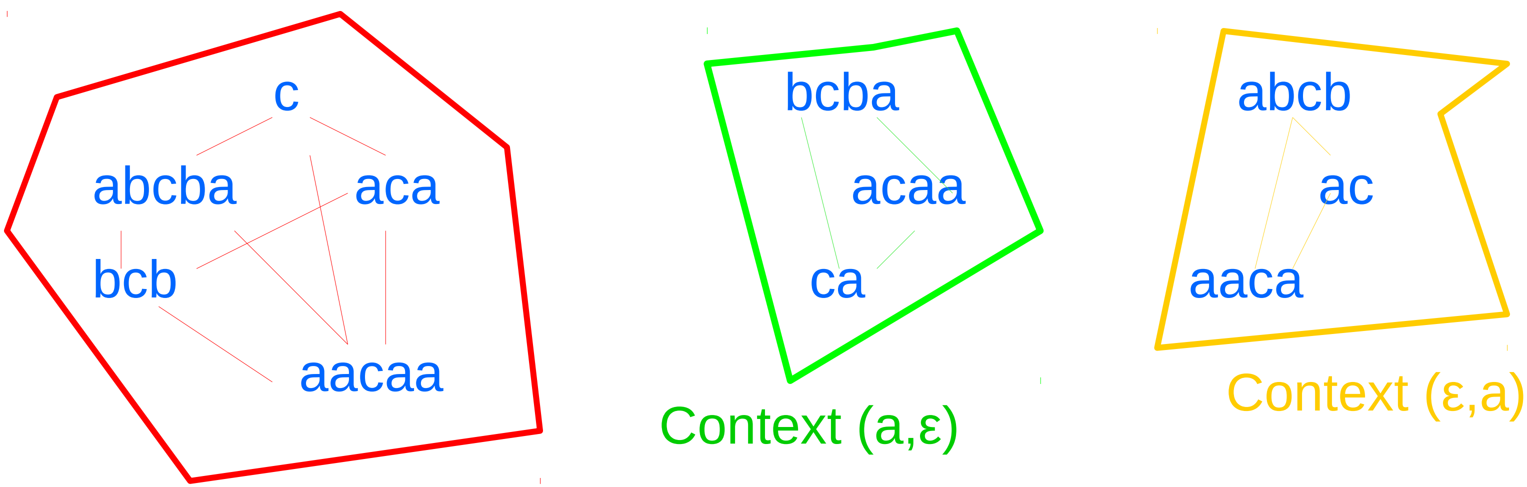
aacaa

a b ab abcb …

bc ba aac caa

Running example

LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

a b ab abcb …

bc ba aac caa

First step: create a non terminal for each component.

[c] (=[aca]=[abcba]=[bcb]=[aacaa])

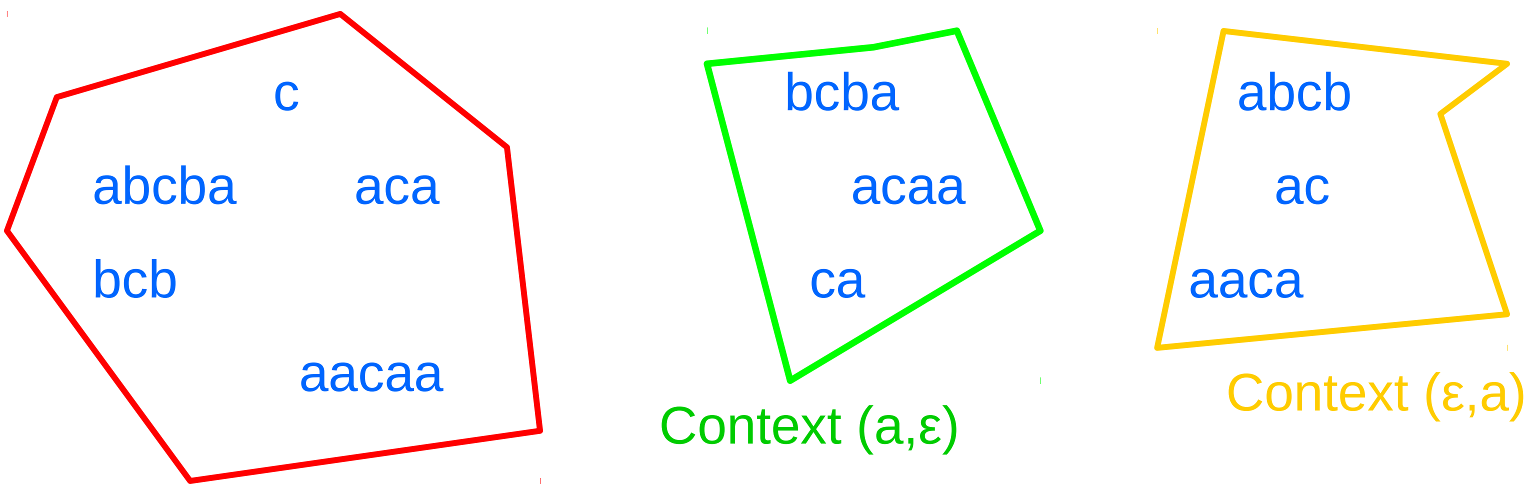
[ca] (=[bcba]=[acaa])

[ac] (=[abcb]=[aaca])

But also: [ab], [abcb], [bc], [aac], [ba]…

Running example

LS={c;aca;bcb;abcba;aacaa}



Context (ε,a)

Context (a,ε)

Empty context (ε,ε)

a b ab abcb …

bc ba aac caa

Next step: create the rules for the letters of the alphabet.

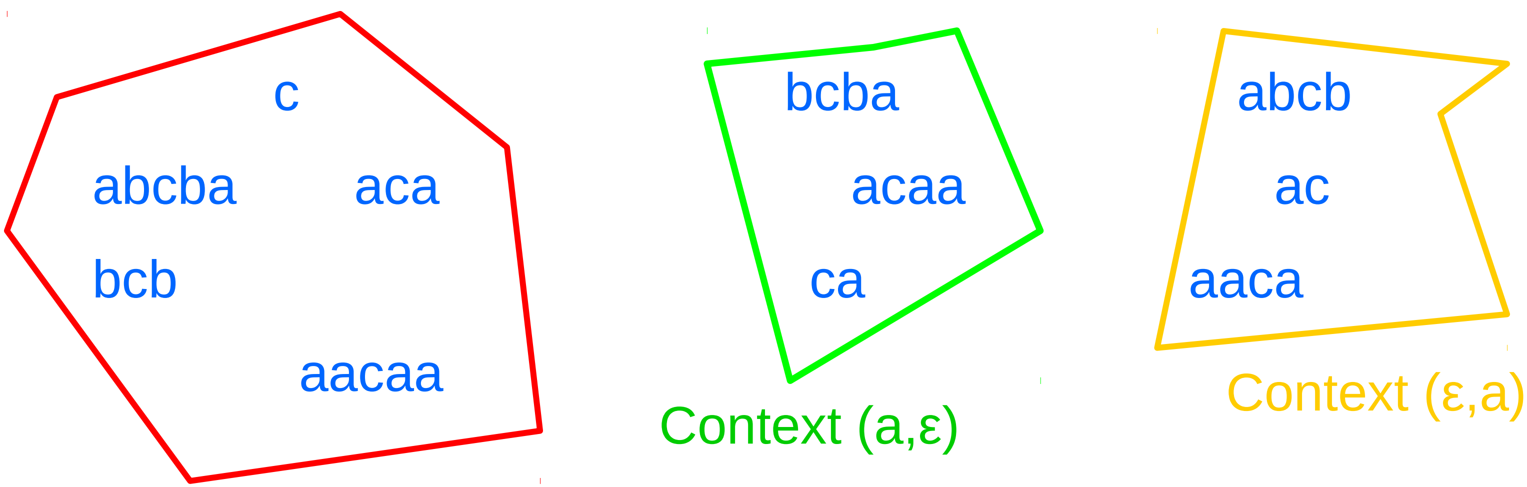
[a] → a

[b] → b

[c] → c

Running example

LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

a b ab abcb …

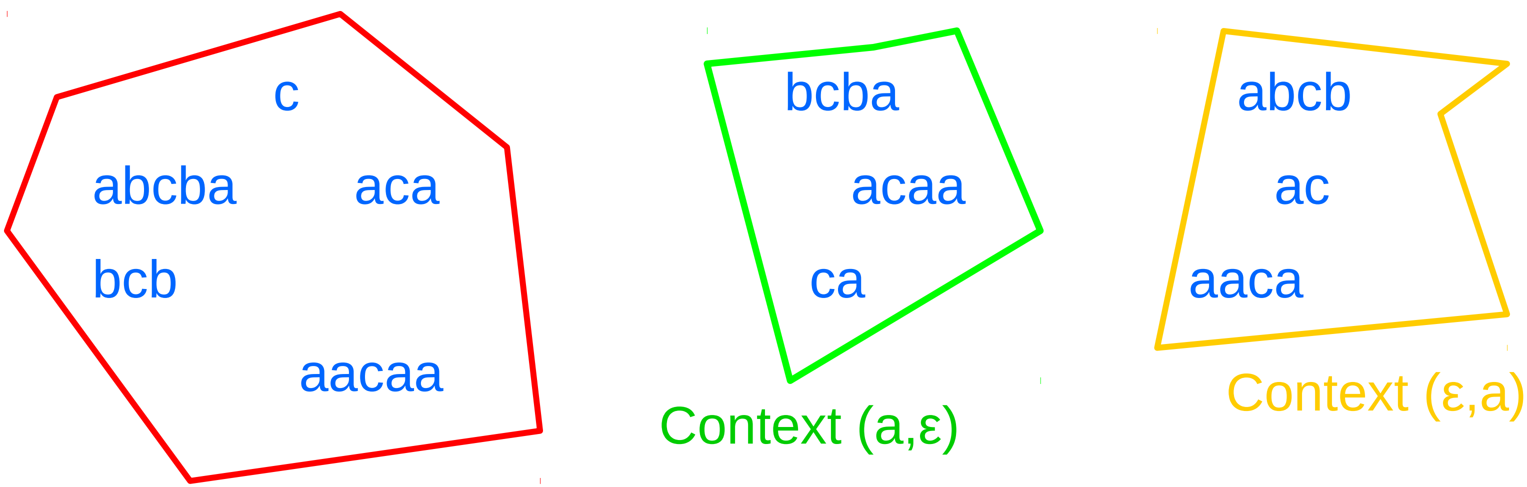
bc ba aac caa

Third step: create the rules for each component.

[w] → [u][v] when uv is a member of the component of w.

Running example

LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

a b ab abcb …

bc ba aac caa

Component (ε,ε):

[c] → [a][bcba], [c] → [ab][cba], c → [abc][ba], [c] → [abcb][a]

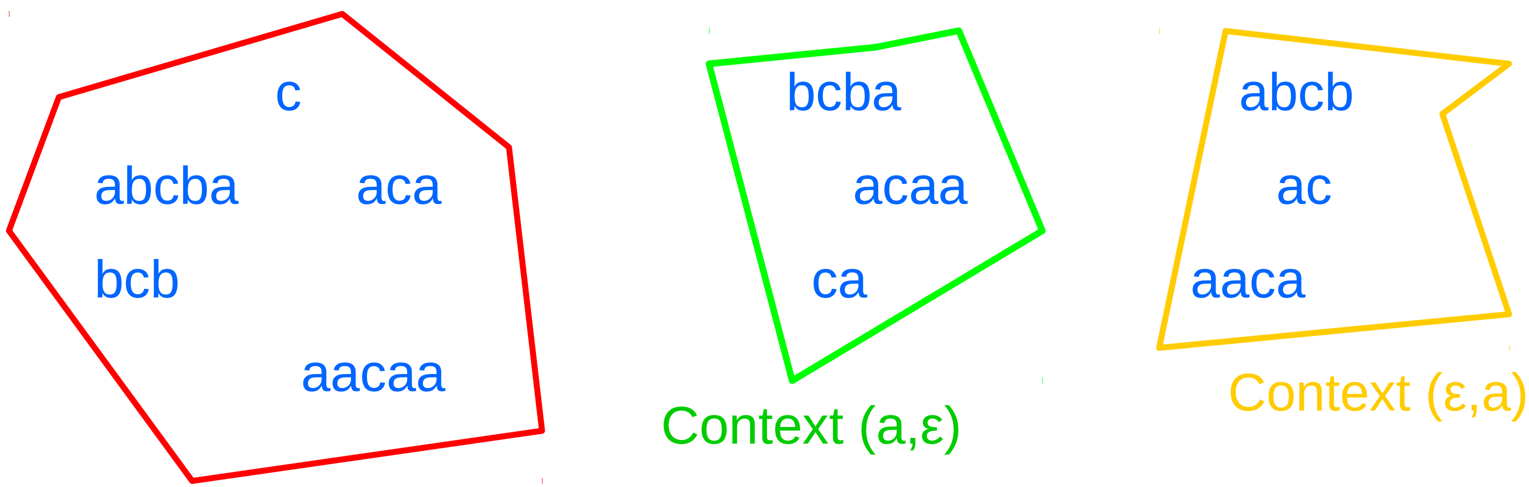
[c] → [a][ca], [c] → [ac][a]

[c] → [b][cb], [c] → [bc][b]

[c] → [a][acaa], [c] → [aa][caa], [c] → [aac][aa], [c] → [aaca][a]

Running example

LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

a b ab abcb …

bc ba aac caa

Component (ε,ε):

[c] → [a] [ca], [c] → [ab] [cba], [c] → [abc] [ba], [c] → [ac] [a]

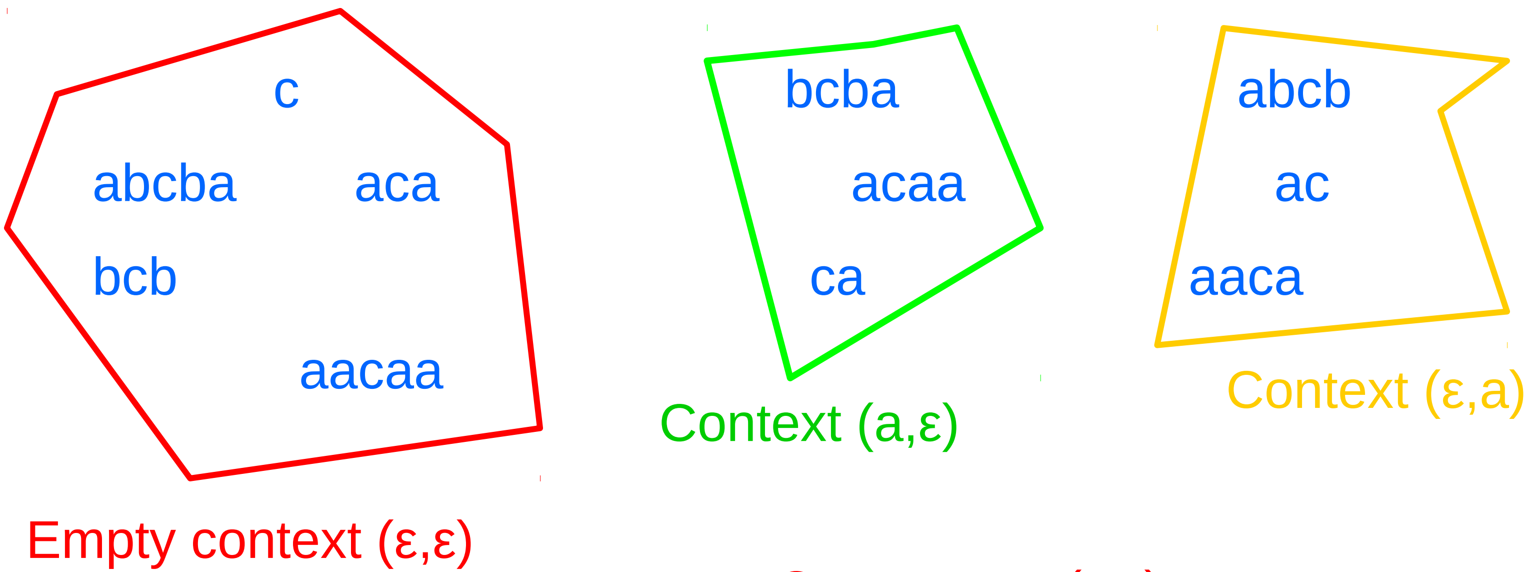
[c] → [a] [ca], [c] → [ac] [a]

[c] → [b] [cb], [c] → [bc] [b]

[c] → [a] [ca], [c] → [aa] [caa], [c] → [aac] [aa], [c] → [ac] [a]

Running example

LS={c;aca;bcb;abcba;aacaa}



a b ab abcb …

bc ba aac caa

Component (ε,ε):

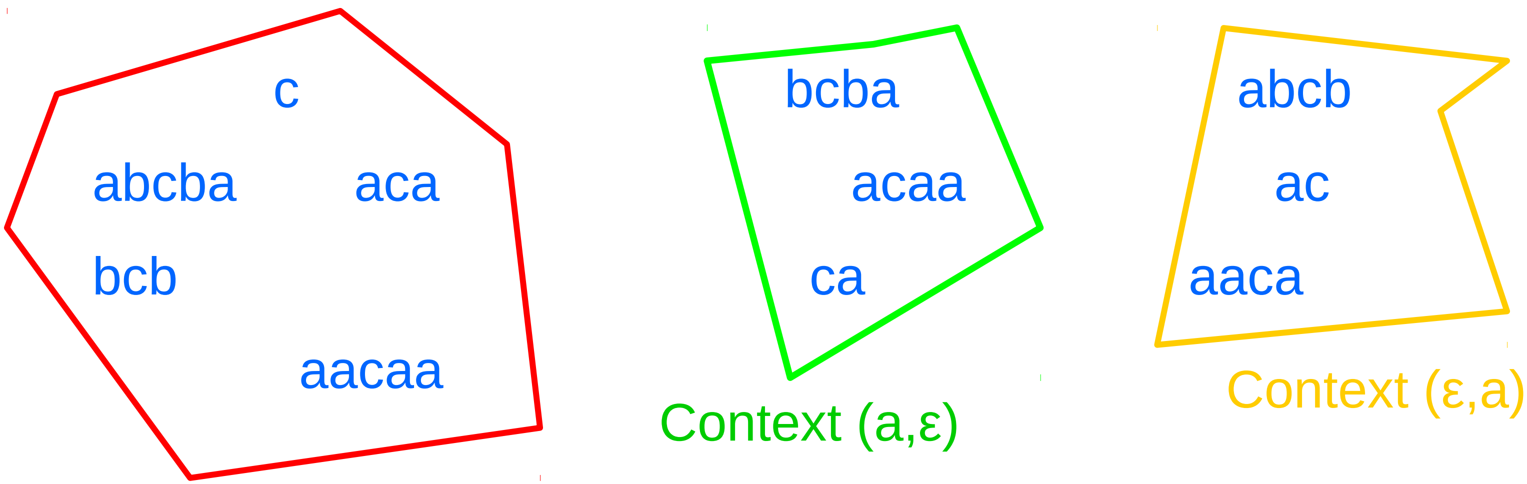
[c]→[a] [ca], [c]→[ac] [a],

[c]→[ab] [cba], [c]→[abc] [ba], [c]→[b] [cb],

[c]→[bc] [b], [c]→[aa] [caa], [c]→[aac] [aa],

Running example

LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

a b ab abcb …

bc ba aac caa

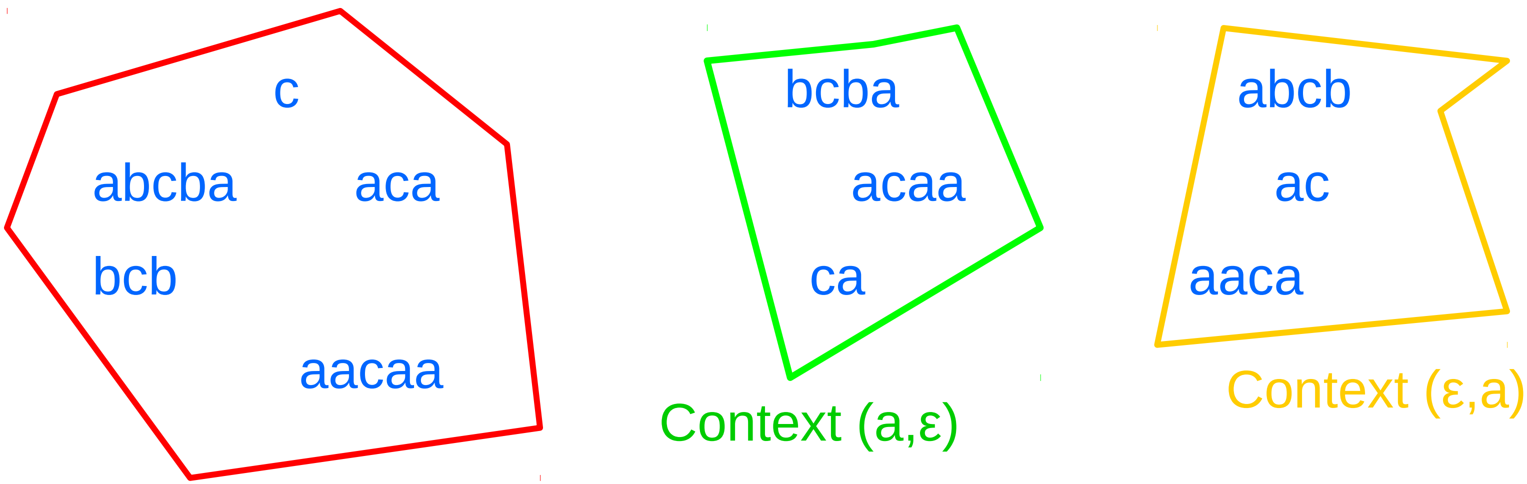
Component (a,ε): (final result)

[ca] → [c] [a], [ca] → [ac] [aa]

[ca] → [b] [cba], [ca] → [bc] [ba], [ca] → [a] [caa]

Running example

LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

a b ab abcb …

bc ba aac caa

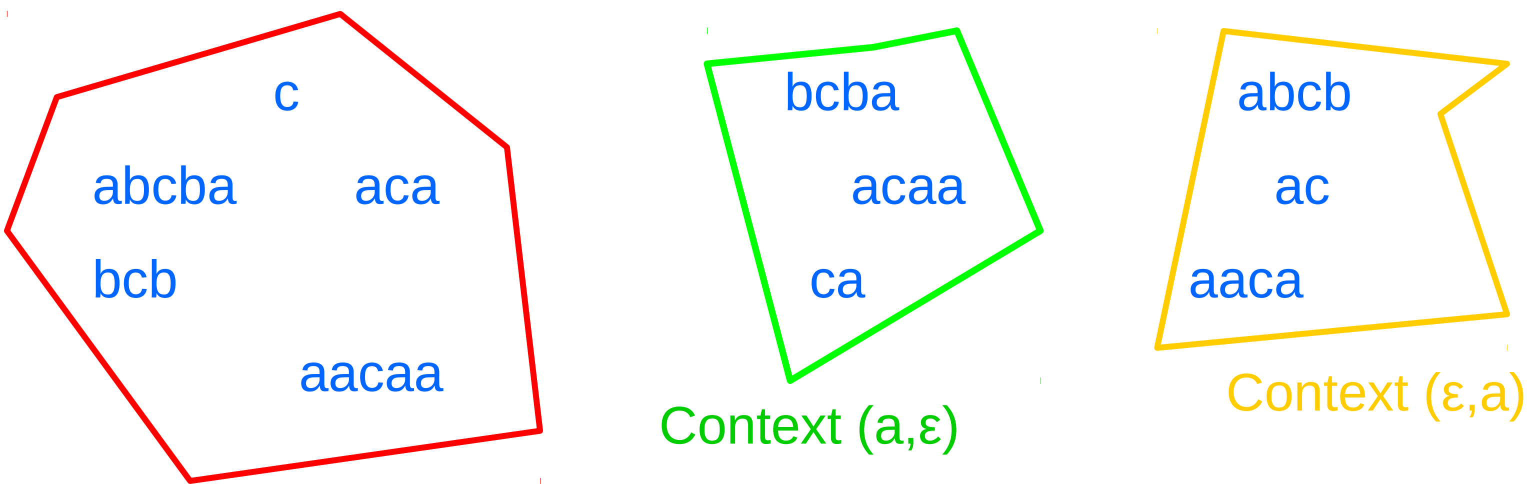
Component (ε,a): (final result)

[ac] → [a] [c], [ac] → [aa] [ca]

[ac] → [ab] [cb], [ac] → [abc] [b], [ac] → [aac] [a]

Running example

LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

a b ab abcb …

bc ba aac caa

Components of one string:

[ab] → [a][b], [aac] → [a][ac], [aac] → [aa][c], [aa] → [a][a] …

Running example

LS={c;aca;bcb;abcba;aacaa}

Outputed grammar: G=< {a,b,c}, V, P, [c] >

where P = {[a] → a, [b] → b,

[c] → [a][ca] | [ac][a] | c, [ca] → [c][a], [ac] → [a][c],

[c] → [ab][cba] | [abc][ba] | [b][cb] | [bc][b] | [aa][caa] | [aac][aa]

[ca] → [ac][aa] | [b][cba] | [bc][ba] | [a][caa]

[ac] → [aa][ca] | [ab][cb] | [abc][b] | [aac] [a]

[ab] → [a][b]

[aac] → [a][ac] | [aa][c]

[aa] → [a][a]

… }

We can show that this grammar generates the language of palindromes with a center marked.

Learning Result

- The algorithm identifies polynomially in the limit the class of context-free substitutable languages.
- The polynomial bounds are on
  - Computation: it takes a polynomial time in the size of the learning sample to run the algorithm.
  - Data: for each substitutable language, there exists a characteristic sample whose *cardinality* is polynomial in the size of the target.

Substitutable context-free and 0-reversible regular languages

- A regular language is 0-reversible if whenever *uw* and *vw* are in the language then *ux* is in the language iff *vx* is in the language [Angluin, 82].
- Substitutable languages are the (context-free) exact analogue of 0-reversible languages (regular).

Direct Extensions

- This work [Clark & Eyraud, 05, 07] generated different extensions
  - Identification of k-l substitutable languages [Yoshinaka, 08]
  - PAC-learning of unambigous NTS languages [Clark, 06], of subclasses of CFG [Shibata&Yoshinaka, 16]
  - Local substitutable languages [Coste, Garet & Nicolas, 12]
  - Substitutable tree languages [Kasprzik & Yoshinaka, 11]
  - Substitutable graph languages [Eyraud, Janodet, Oates, 12&16]
  - Identification with the help of an oracle of congruential context-free laguages [Clark, 10], conjunctive grammars [Yoshinaka, 15], Parallel Multiple CF grammars [Clark & Yoshinaka, 14]
- Heuristics have preceded theory [Harris, 54], [Brill et al., 90] [Adriaans, 99], [van Zaanen, 00] [Klein & Manning, 02], ...

Substitutable Languages and Natural Languages

- Natural languages are obviously more complex, but substitutable ones can give nice insights into some linguistic disputes.
- Ex.: Auxiliary fronting in polar questions

Sentences like '*Is the man who is hungry ordering dinner?*' are unlikely to be present in a child environment.

However, it has been shown that children are quickly able to identify it as correct and reject the wrong sentence '*Is the man who hungry is ordering dinner?*'

Used as a clue in defence of the theory of innate knowledge of native language.

A (very) simple example

S = { the man who is hungry asked a beer.

the man asked a beer.

the man is hungry.

the man is ordering dinner.

is the man hungry? }

(+) is the man who is hungry ordering dinner?

(-) is the man who hungry is ordering dinner?

- Our algorithm can identifies correct structure from incorrect one without any example on this particular structure in the sample.

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- Conclusion

Congruencial classes

- Recall: Given a language L, [u] is the congruence class of *u*, that is the set of all substrings *v* such that $u \equiv_L v$
- If the language is not regular : an infinite number of congruence classes
- However, substitutable (and others) can be represented by a finite number of them

Prime congruence classes

- Two particular congruence classes:
  - The unit: [ε]
  - The zero: 0 = {u : for all (l,r), lur is not in L}
- A congruence class X is **prime** if it is non-zero and non-unit and for any two congruence classes Y, Z such that X = Y · Z then either Y or Z is the unit.
  If a non-zero non-unit congruence class is not prime then we say it is composite.

Prime congruence classes

- Example: $L=\{a^n cb^n : n \geq 0\}$
  - $[\varepsilon]=\{\varepsilon\}$ and $0=\Sigma^* ba \Sigma^*$: not prime by definition,
  - L : prime (because $L=[c]=[aca]\neq[a][ca]=\{a^n cb^n : n \geq \mathbf{1}\}$)
  - $[a] = \{a\}$ and $[b] =\{b\}$: both prime
  - $[a^i]=\{a^i\}=[a][a^{i-1}]$ with $i>1$: not prime (same for b)
  - $[a^i c] = \{a^{i+j} cb^j : j \geq 0\}$ and $[cb^i] = \{a^j cb^{j+i} : j \geq 0\}$: not prime
- Note: $L = \{ab\}$ has 5 congruence classes: $[a]$, $[b]$, $[ab]$, $[\varepsilon]$ and 0. The first 4 are all singleton sets. $[a]$ and $[b]$ are prime but $[ab] = \{ab\} = [a][b]$, and so L is not prime.

L_{sc}

- L_{sc} : the set of all languages which are substitutable, non-empty, do not contain ε, and have a finite number of prime congruence classes.
- Example: $\{a^n c b^n : n \geq 0\}$
- Counter-example: $L=\{c^i b a^i b : i>0\} \cup \{c^i d e^i d : i>0\}$
  - CF and substitutable
  - But for all i, $C_L(ba^ib) = \{(c^i, \varepsilon)\} = C_L(de^id)$
  - Infinite number of classes $[ba^ib]=[de^id]=\{ba^ib, de^id\}$, each of which is prime.

Prime decomposition

- A prime decomposition of a congruence class X is a finite sequence of one or more prime congruence classes $\alpha = \langle X_1 , \ldots , X_k \rangle$ such that $X = X_1 X_2 \ldots X_k$
- Lemma: Every non-zero non-unit congruence class of a language in L_{sc} has a unique prime factorisation

Correct rules

- Correct production: $[\bar{\alpha}] \rightarrow \alpha$ where α is a sequence of at least 2 primes and $[\bar{\alpha}]$ is a prime congruence class.

  A correct lexical production is one of the form $[a] \rightarrow a$ where $a \in \Sigma$, and $[a]$ is prime.

- Ex: $L = \{a\ n\ cb\ n \mid n \geq 0\}$. Primes: [a], [c], [b].

  The correct lexical productions are the three obvious ones $[a] \rightarrow a$, $[b] \rightarrow b$ and $[c] \rightarrow c$.

  The only correct productions have [c] on the left hand side, and are $[c] \rightarrow [a][c][b]$, $[c] \rightarrow [a][a][c][b][b]$ and so on.

Too long rules

- We say that a sequence of primes α is pleonastic (too long) if $\alpha = \gamma\beta\delta$ for some γ, β, δ, which are sequences of primes, such that $|\gamma| + |\delta| > 0$, $[\beta]$ is a prime, and $|\beta| > 1$
- A rule is too long if its right handside is too long. It is valid otherwise

Canonical grammar

- Lemma: If $L \in L_{sc}$ then there are a finite number of valid productions
- Let L in L_{sc}. Its (unique) canonical grammar $G_*(L)$:
  - Non-terminals: the prime congruence classes of L, together with an additional symbol S (the start symbol).
  - Productions:
    - the single production containing the start symbol: $S \rightarrow \alpha(L)$, where $\alpha(L)$ be the unique prime decomposition of L.
    - All valid productions
    - The production $[a] \rightarrow a$, for each terminal symbol *a* that occurs in the language
- Theorem: the language of $G_*(L)$ is L

Learning result

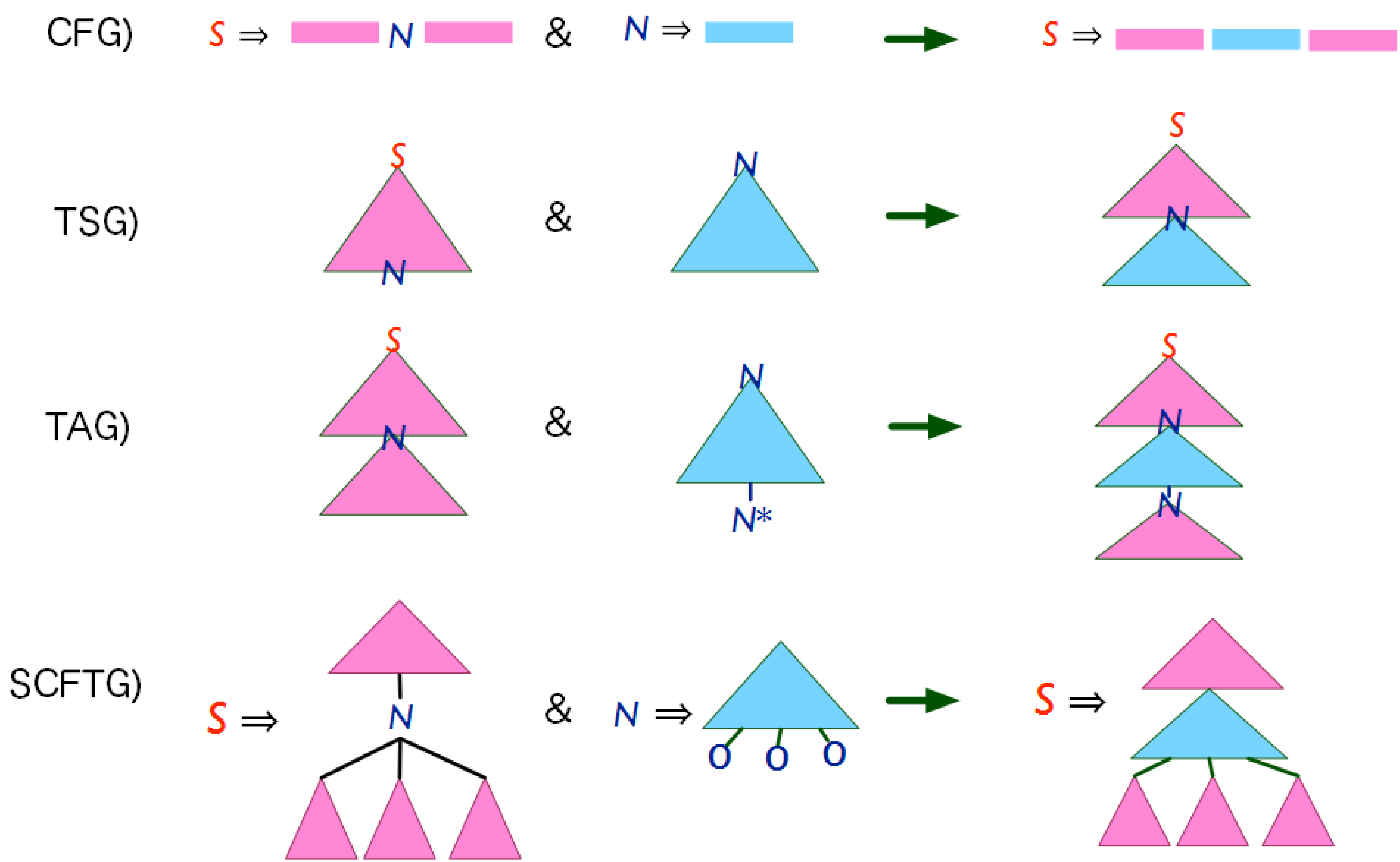
- The previous algorithm can be adapted to output only canonical grammars: it identifies in the limit the class L_{sc}
- Moreover, it learns exactly the target grammar (=language + structure): strong learning
- Corrolary: it learns trees from strings!

Outline

- Introduction
- Learning Substitutable languages
- Prime congruence classes
- **Extension to tree and graph grammars**
- A dual approach
- Conclusion

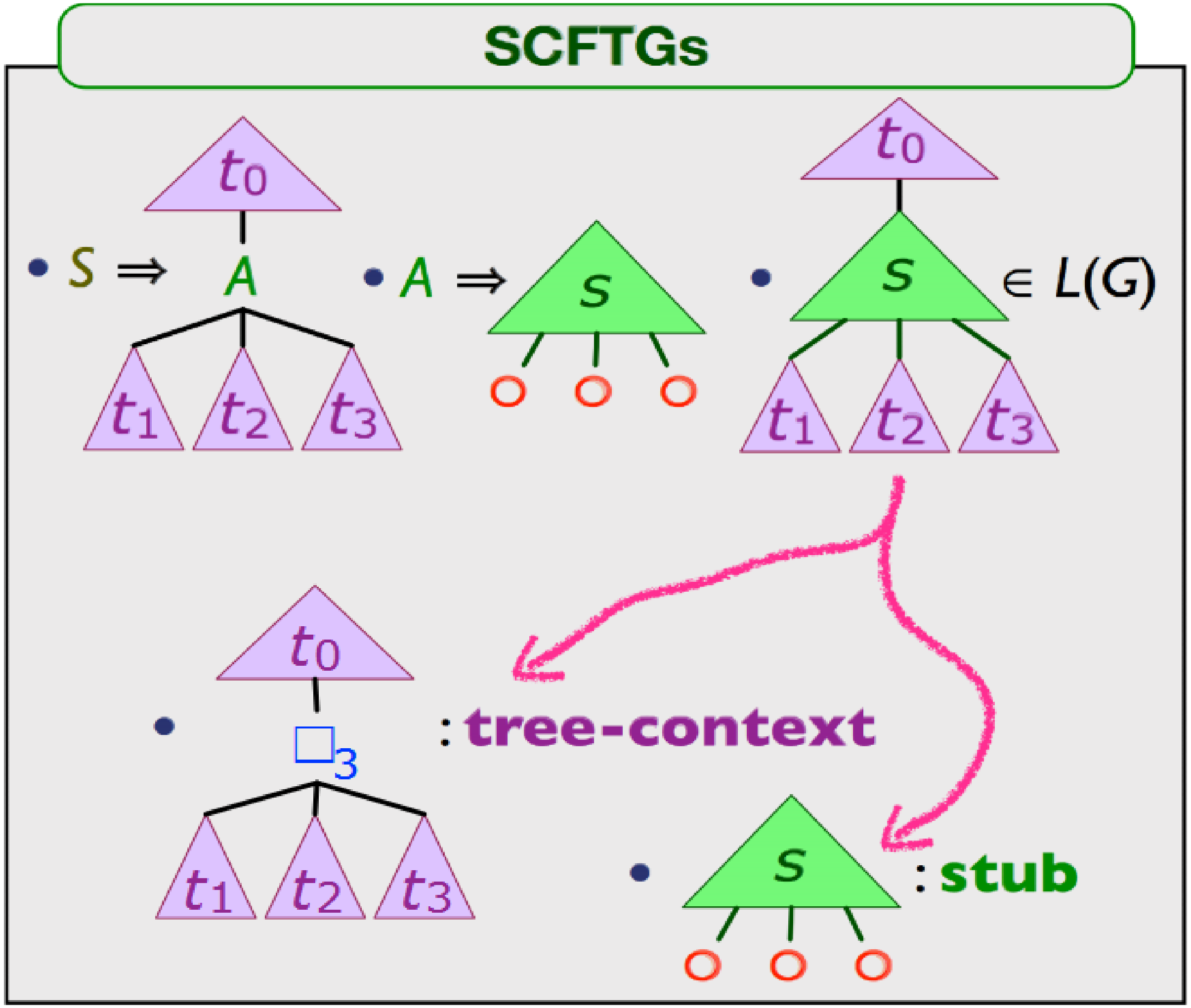
Extension to tree

- Instead of strings the data are trees
- The grammars are Simple Context Free Tree Grammar



Extension to tree

- All we need is the transposition of the notions of context, subtree and a gluing mechanism.



Distributional learning of tree languages

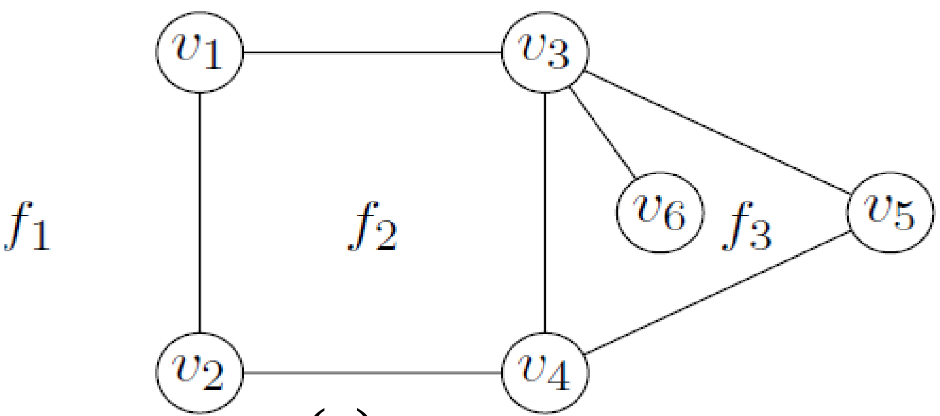
- [Kasprzik & Yoshinaka, 11]:
  - r-substitutable context-free tree languages are efficiently identifiable in the limit from positive tree examples.
  - r-SCFTG with p-finite environment are identifiable from positive presentation using a membership oracle.
  - The algorithms are simple adaptation to trees of the ones for the strings.

Extension to graphs

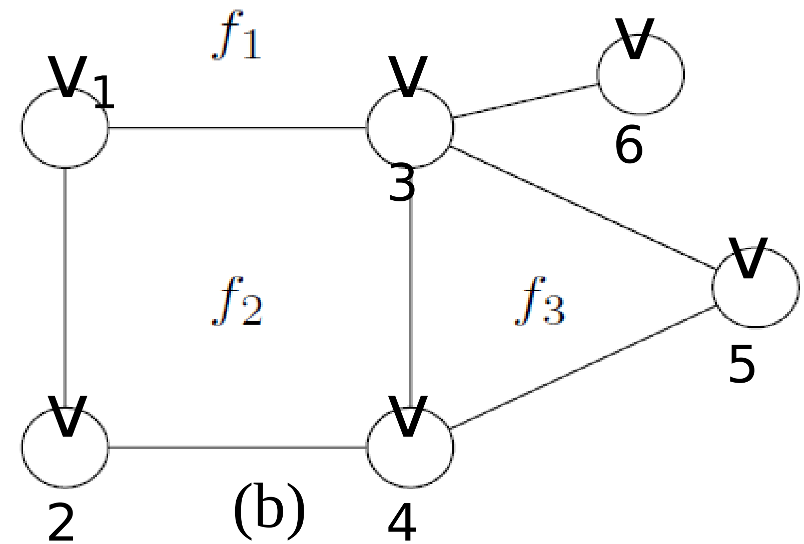
- Again, we need to define what a context is, what a subgraph is, and how to glu them together.
- But we also need to restrict ourselves as general graphs are too complex:
  - We need tractable isomorphism (and sub-isomorphisme)
  - We need a grammar formalism where it is polynomially doable to test whether a given graph is in the language
- Plane graphs are good candidate: polynomial decidable sub-isomorphism.

Plane Graph

- Embeddings of planar graph in the plan:



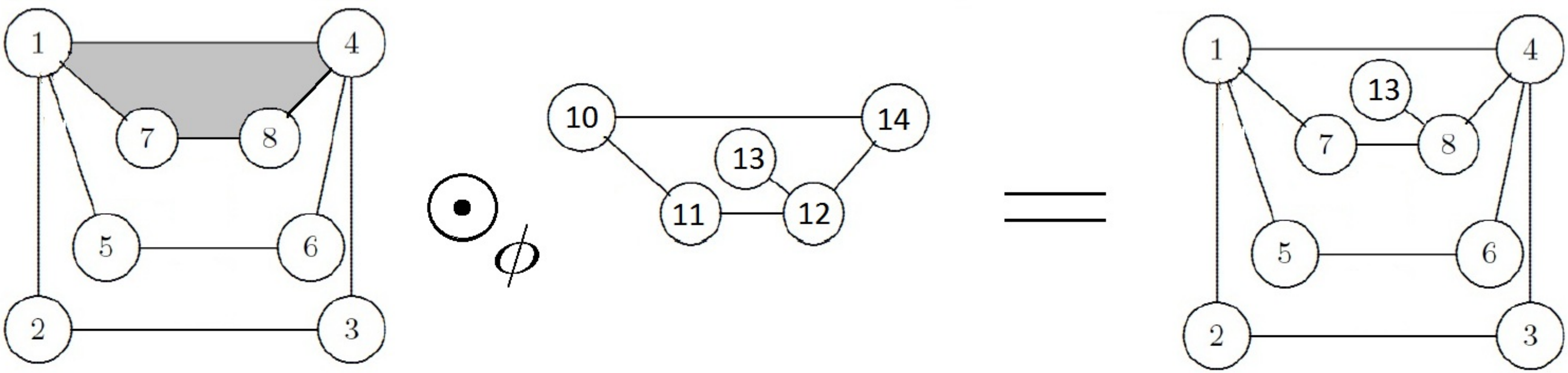
(a)



(b)

- Class of isotopy

Substitutability



- In a substitutable plane graph language, whenever two plane graphs appear in the same context once, they share the same set of contexts.

Learning result

- [Eyraud, Janodet, Oates, 16]: Substitutable plane graph languages are identifiable in the limit from examples of the language

  - Promising research
  - Languages are close under isomorphism.
  - First non-trivial class of graph grammars to be learnable

  - Algorithm is not efficient
  - Hard to extend to more complex kind of graphs

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Distributional Learning

- A new familly of approaches to handle grammar induction:
  - "Observe, model, exploit the relation between substrings and contexts"
- Primal (ex: substitutable): a non-terminal represent a (set of) string that appear in the same contexts: if [u] $\rightarrow^*$ v then $C_L(u) = C_L(v)$
- Dual: a non-terminal represent a (set of) context and generates only strings that appear in this context: if [(l,r)] $\rightarrow^*$ u then $u \in \{w : lwr \text{ in } L\} = S_L(l,r)$

An example

- $L = \{a^n b^n : n \geq 0\}$
- Observation table:

	(ε,ε)	(a,ε)	(ε,b)	(a,b)
ε	yes	no	no	yes
a	no	no	yes	no
b	no	yes	no	no
ab	yes	no	no	yes
aab	no	no	yes	no
abb	no	yes	no	no
aabb	yes	no	no	yes

An example

- $L = \{a^n b^n : n > 0\}$
- Observation table:

	(ε,ε)	(a,ε)	(ε,b)	(a,b)
ε	1	0	0	1
a	0	0	1	0
b	0	1	0	0
ab	1	0	0	1
aab	0	0	1	0
abb	0	1	0	0
aabb	1	0	0	1

An example

- L = $\{a^n b^n : n>0\}$
- Observation table:

	(ε,ε)	(a,ε)	(ε,b)	(a,b)
ε	1	0	0	1
a	0	0	1	0
b	0	1	0	0
ab	1	0	0	1
aab	0	0	1	0
abb	0	1	0	0
aabb	1	0	0	1

- Primal: similar lines (or similar parts of different lines) **may** correspond to the same non-terminal

An example

- $L = \{a^n b^n : n > 0\}$
- Observation table:

	(ε,ε)	(a,ε)	(ε,b)	(a,b)
ε	1	0	0	1
a	0	0	1	0
b	0	1	0	0
ab	1	0	0	1
aab	0	0	1	0
abb	0	1	0	0
aabb	1	0	0	1

- Dual: similar columns **may** correspond to the same non-terminal

An example

- $L = \{a^n b^n : n > 0\}$ but we only "see" the sample S={ab, aabb}
- Observation table:

	(ε,ε)	(a,ε)	(ε,b)	(a,b)
ε	?	?	?	1
a	?	?	1	?
b	?	1	?	?
ab	1	?	?	1
aab	?	?	1	?
abb	?	1	?	?
aabb	1	?	?	?

- Ask an oracle for the missing information

An example

- $L = \{a^n b^n : n>0\}$ but we only “see” the sample S={ab, aabb}
- Real observation table:

	(ε,ε)	(a,ε)	(ε,b)	(a,b)	(aa,ε)	(aab,ε)	(ε,bb)	(ε,abb)	(aa,b)	(a,bb)
ε				+						
a			+					+		+
b		+				+			+	
ab	+			+						
aa							+			
bb					+					
aab			+							
abb		+								
aabb	+									

An example

- L = $\{a^n b^n : n>0\}$ but we only “see” the sample S={ab, aabb}
- Real observation table:

Context set

Kernel

	(ε,ε)	(a,ε)	(ε,b)	(a,b)	(aa,ε)	(aab,ε)	(ε,bb)	(ε,abb)	(aa,b)	(a,bb)
ε				+						
a			+					+		+
b		+				+			+	
ab	+			+						
aa							+			
bb					+					
aab			+							
abb		+								
aabb	+									

Learning principle

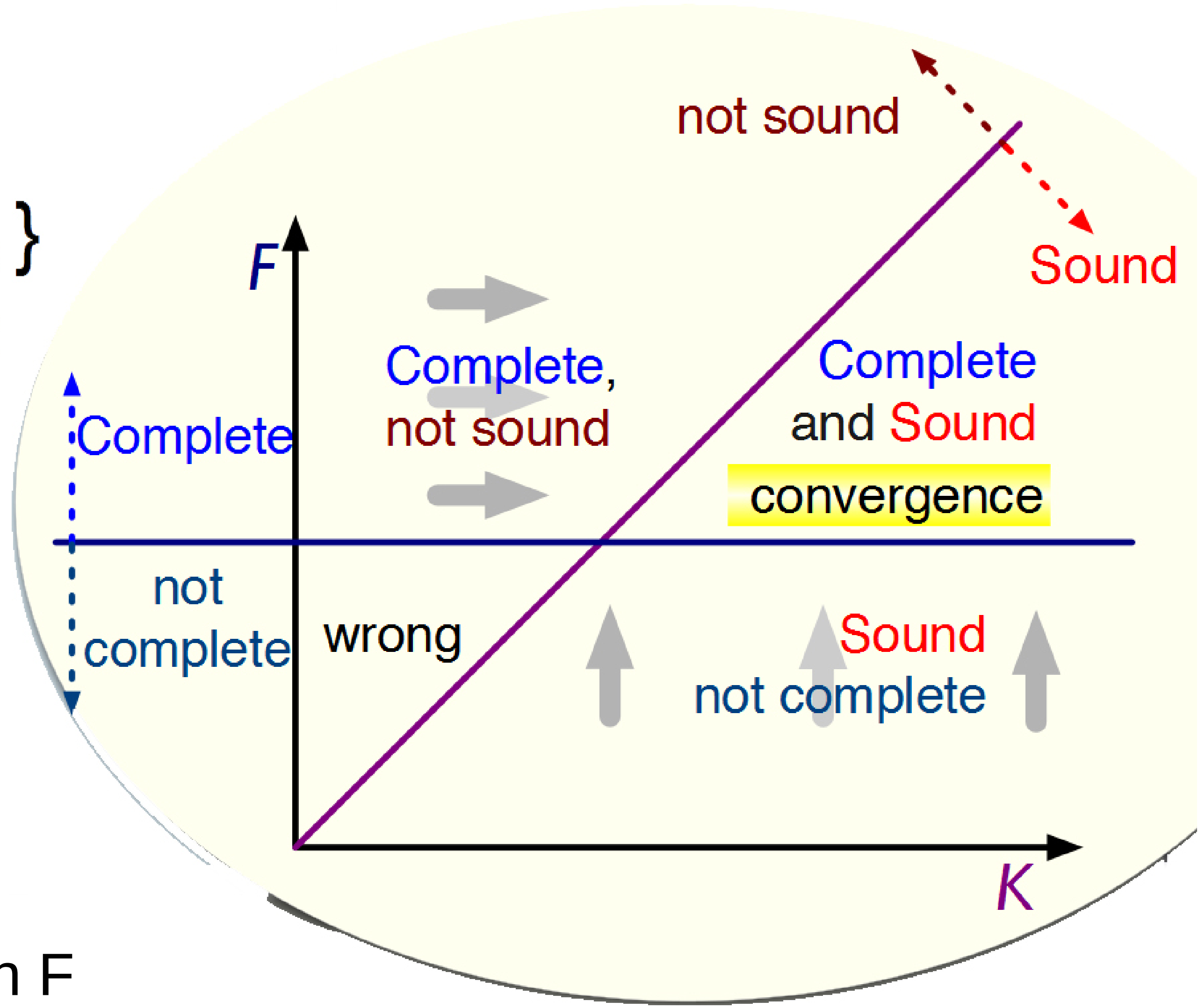
- Problem: if the language is not regular, there exists a non-finite number of syntactic congruence classes.
- What we need is to restrict ourselves to classes of languages that are representable by finitely many congruence classes
- Then we may observe these classes in the completed observable table constructed from a finite sample:
  - Either by considering similar rows (primal)
  - Or by considering similar column (dual)

Finite Context Property (dual)

- A CFG $G = < \Sigma, N, S, P >$ has the (one) Finite Context Property iff every $A \in N$ admits a characterizing context (l,r) such that $S_L(l,r)=\{v: N \rightarrow^* v\}$
- Examples: all regular languages, parenthesis languages, ...
- With enough data, one column of the observation table corresponds exactly to each non-terminal of the target grammar.
  - Adding new columns add new non-terminals (thus new rules)
  - Adding new lines remove incorrect rules

Learning in the dual

Let $D := K := F := \emptyset$;
For $n = 1,2,3,\ldots$
  let $D := \{u_1, u_2, \ldots, u_n\}$
  If $D \not\subseteq L(G(F,K))$ then
    let $F :=$ Con(D)
  End if
  let $K :=$ Sub(D)
  output $G(F,K)$
End for



Grammar creation:
Non-terminals: [(l,r)], (l,r) in F
Rules: [(l,r)] → a if *lar* in L
[(l,r)] → [(l_1,r_1)][(l_2,r_2)] if for all w_1, w_2 in K s.t. $l_1w_1r_1$ and $l_2w_2r_2$ in L we have lw_1w_2r in L

Learning results

- [Clark, 10 ; Yoshinaka, 11] 1-FCP (and 1-FKP) classes are identifiable in the limit from examples using a membership query with
  - An update time polynomial in the size of the sample
  - An number of queries polynomial in the size of the target grammar.
- Extended to k-FCP (and k-Finite Kernel Property and k-Finite Distributional Property where each non-terminal has either a characteristic context or characteristic string).
- Extended to other classes of grammars: multiple CFG and parallel CFG.
- Extended to PAC-learning results

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Overall results (in 2012...)

		String formalisms		Tree form.	Graph form.
		CFG	MCFG	SCFTG	PGG
Primal	Substitutable	Clark & Eyraud '05,'07	Yoshinaka '09	Kasprzik & Yoshinaka '11	Eyraud, Janodet & Oates '12
	Congruential	Clark '10	Yoshinaka & Clark '10		
	Finite Kernel Property	Yoshinaka'11	Yoshinaka'10	Kasprzik & Yoshinaka '11	
Dual	Context-deterministic	Shirakawa & Yokomori '93			
	Finite Context Property	Clark, Eyraud & Habrard '08 (CBFG) Clark'10		Kasprzik & Yoshinaka '11	

Next steps

- Restrictions rely on a class of grammars (to make sure the language is representable by a finite number of congruence classes).
- Use of a membership oracle (for the more complex classes).
- Not practical that way but nice proof of concept.

MISC. citations

- John Myhill, 1950, commenting on Bar-Hillel

I shall call a system regular if the following holds for all expressions μ,ν and all wffs φ,ψ each of which contains an occurrence of ν: If the result of writing μ for some occurrence of ν in φ is a wff, so is the result of writing μ for any occurrence of ν in ψ. Nearly all formal systems so far constructed are regular; ordinary word-languages are conspicuously not so.

- Noam Chomsky review of Greenberg (1959)

Let us say that two units A and B are substitutable1 if there are expressions X and Y such that XAY and XBY are sentences of L; substitutable2 if whenever XAY is a sentence of L then so is XBY and whenever XBY is a sentence of L so is XAY (i.e. A and B are completely mutually substitutable). These are the simplest and basic notions. (footnote 3. They are discussed by R. Carnap in The Logical Syntax of Language, 1934)

String Rewriting Rule

- Introduced in 1914 by Alex Thue.
- A string rewriting rule replaces a substring of a string by another substring.
- Example: ab $\rightarrow$ ε

This rule replaces substings ab by ε, i.e. it erases substrings ab.

a<u>ab</u>bab $\rightarrow$ ab<u>ab</u> $\rightarrow$ <u>ab</u> $\rightarrow$ ε

Running Example

- LS={c;aca;bcb;abcba;aacaa} (palindromes with a center marked)

c bcba abcb

abcba aca acaa ac

bcb ca aaca

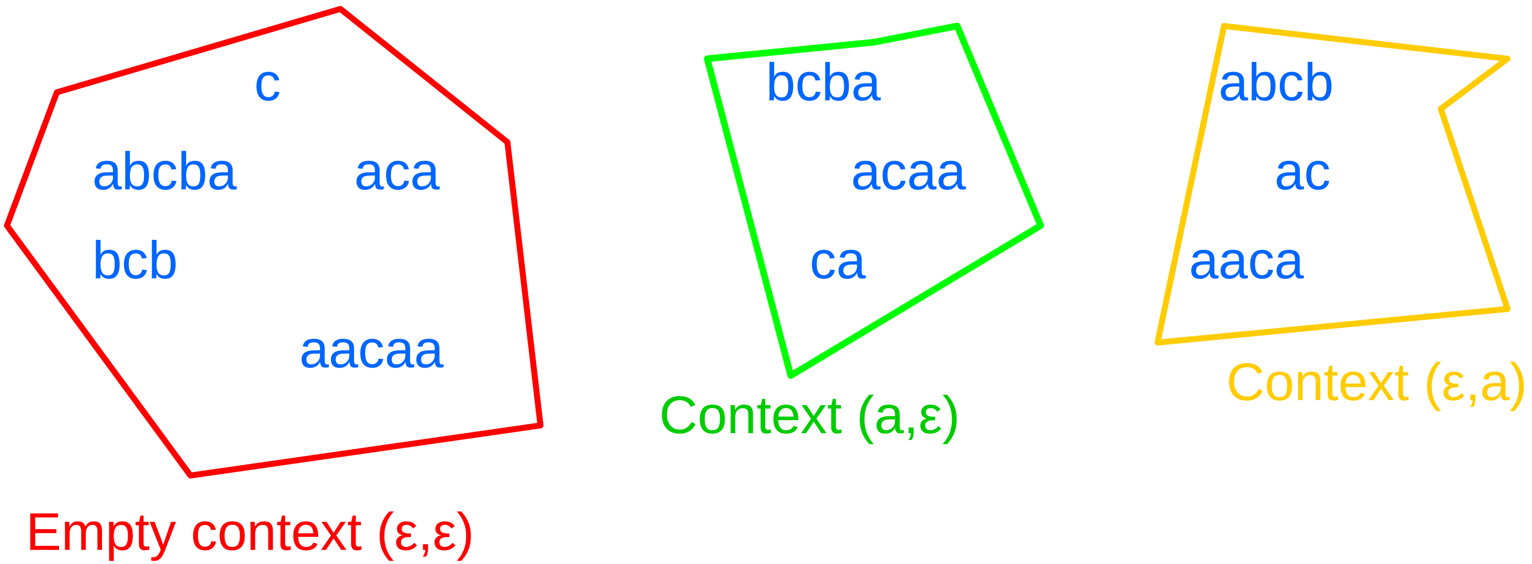
aacaa

a b ab abcb …

bc ba aac caa

Running Example

- LS={c;aca;bcb;abcba;aacaa}

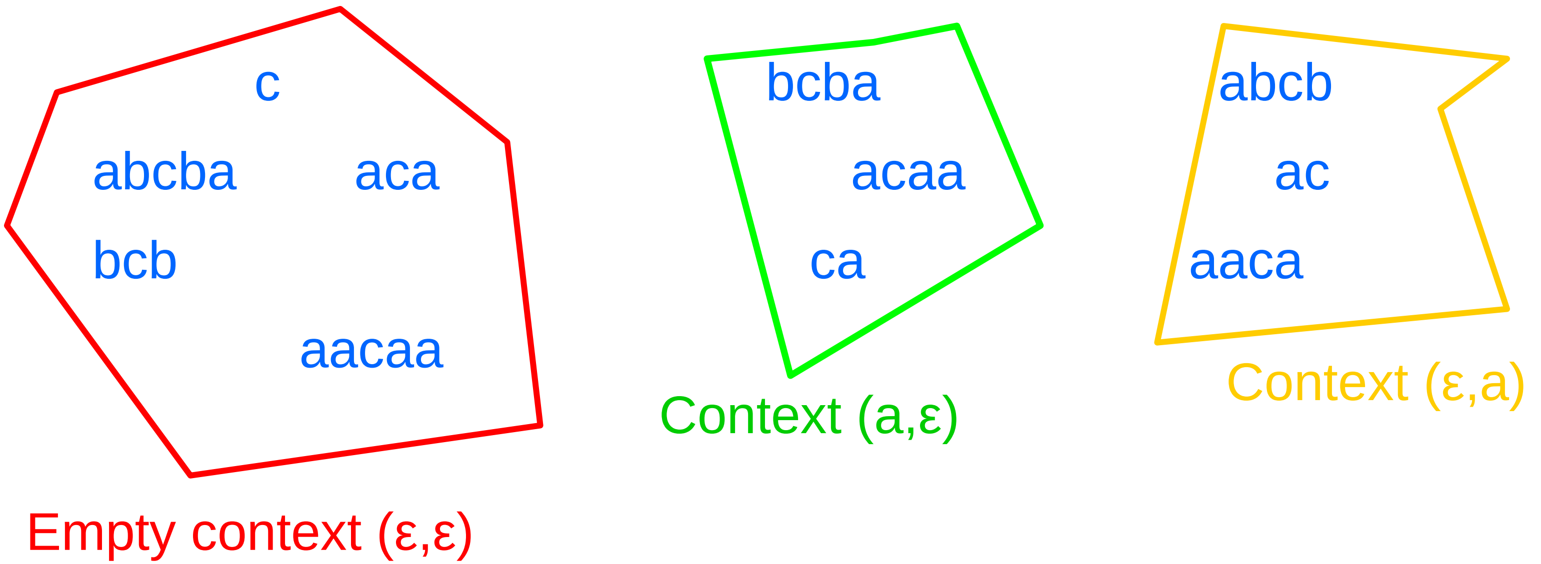


a b ab abcb …

bc ba aac caa

Running Example

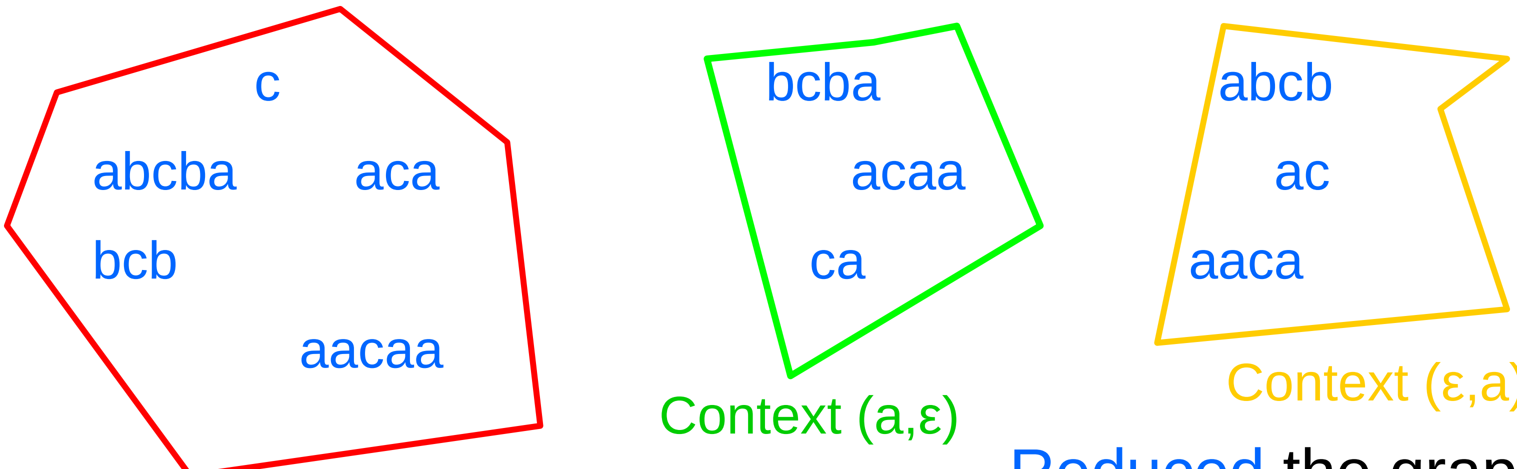
- LS={c;aca;bcb;abcba;aacaa}



Erase component made of a unique element.

Running Example

- LS={c;aca;bcb;abcba;aacaa}



Empty context (ε,ε)

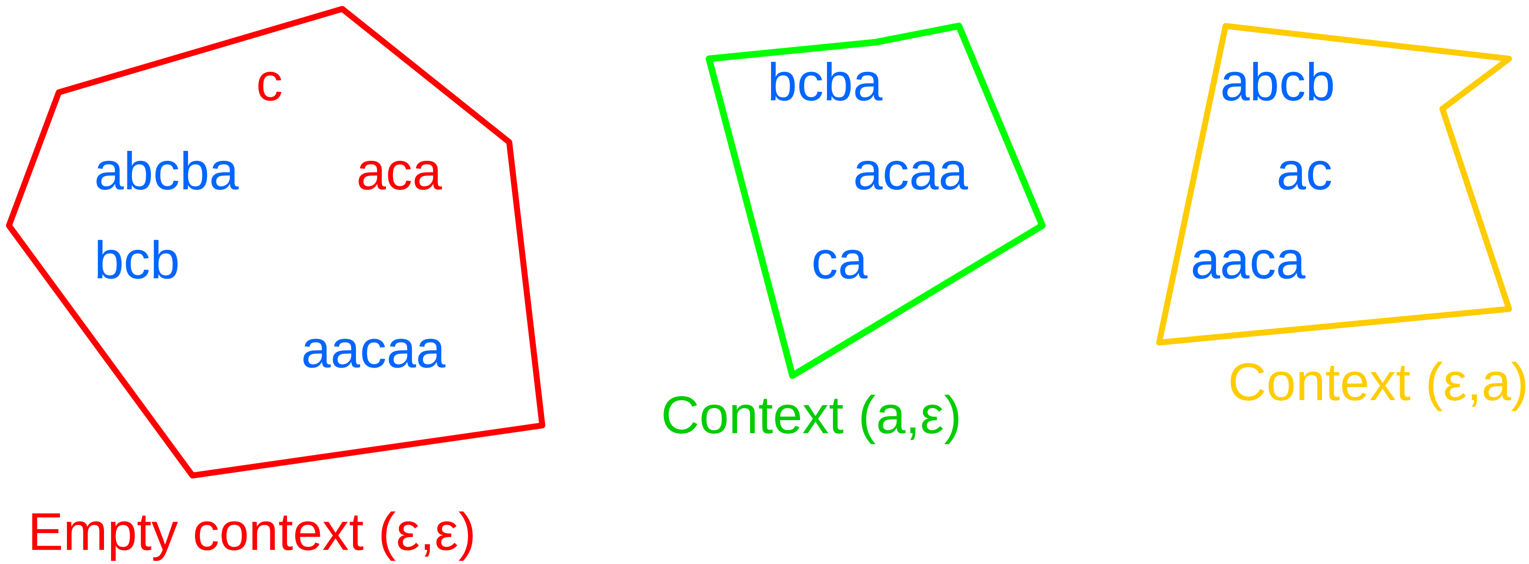
Reduced the graph:

If u and v are in the same component and u > v

Then replace every lur by lvr

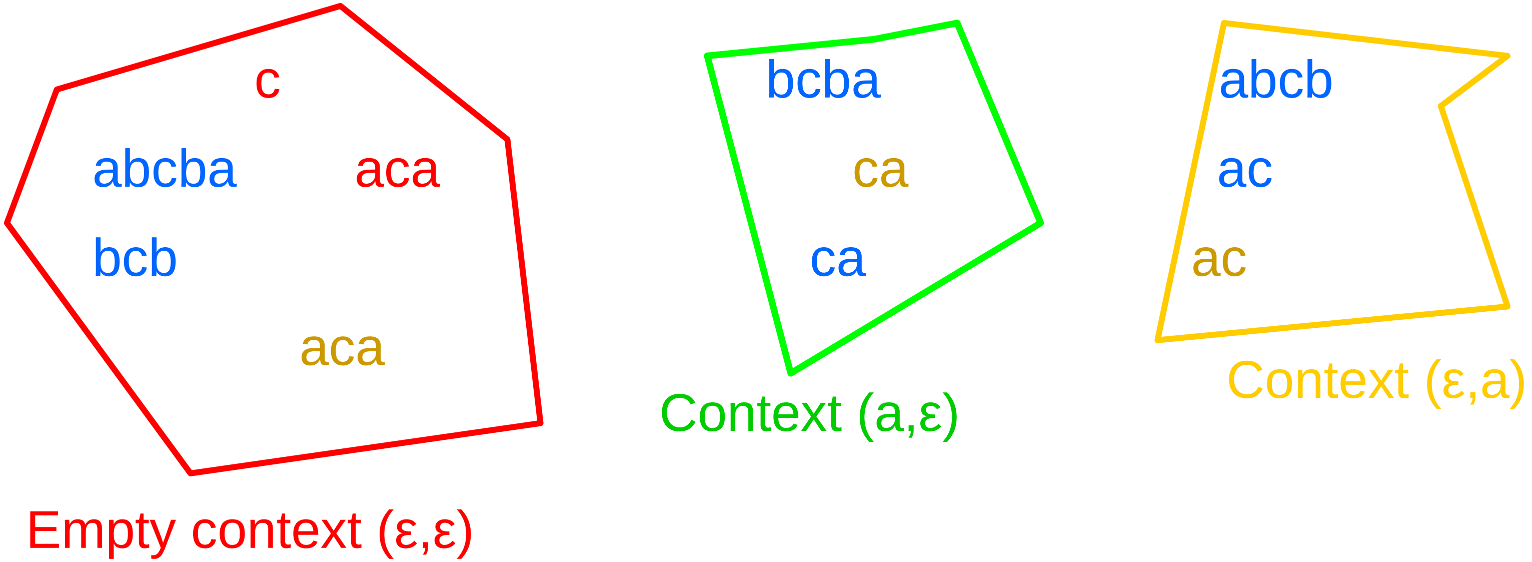
Running Example

- LS={c;aca;bcb;abcba;aacaa}



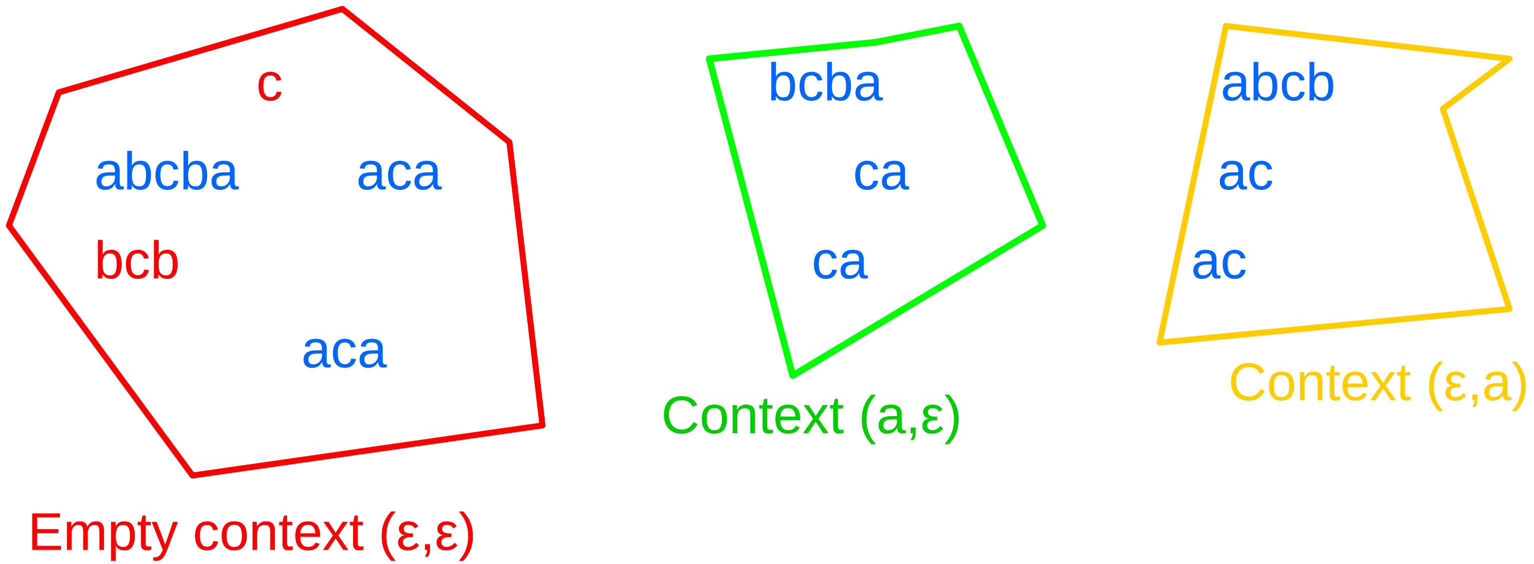
Running Example

- LS={c;aca;bcb;abcba;aacaa}



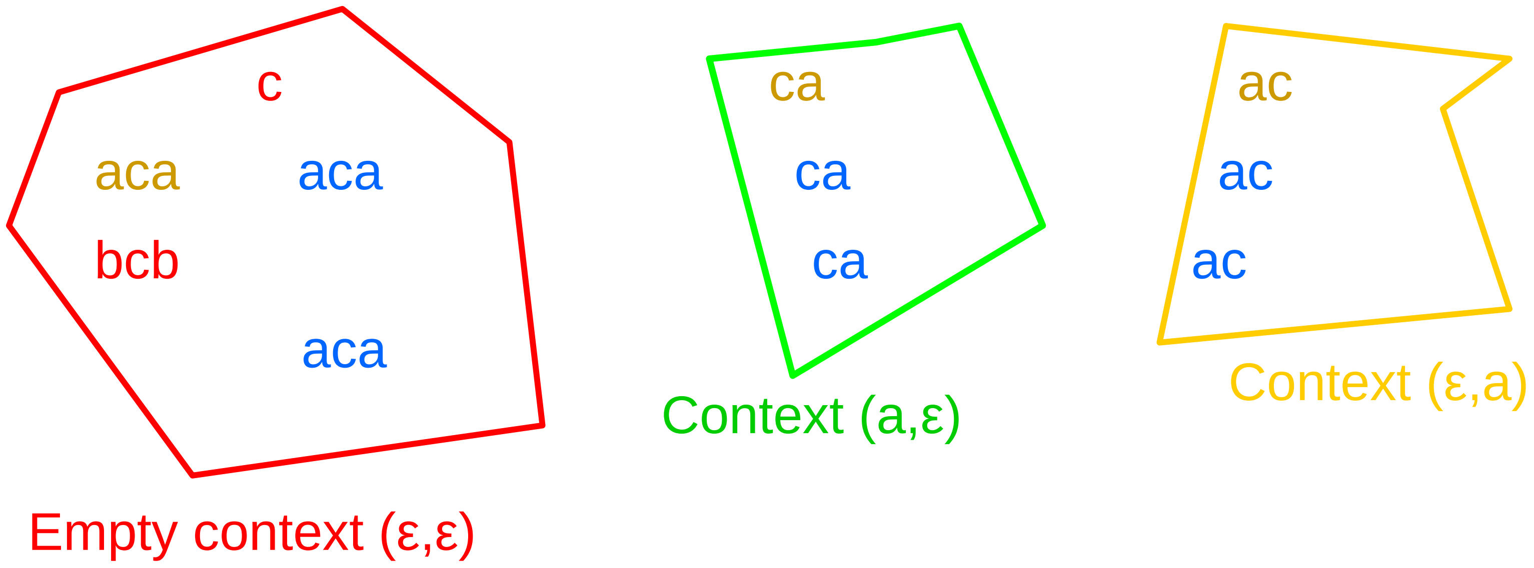
Running Example

- LS={c;aca;bcb;abcba;aacaa}



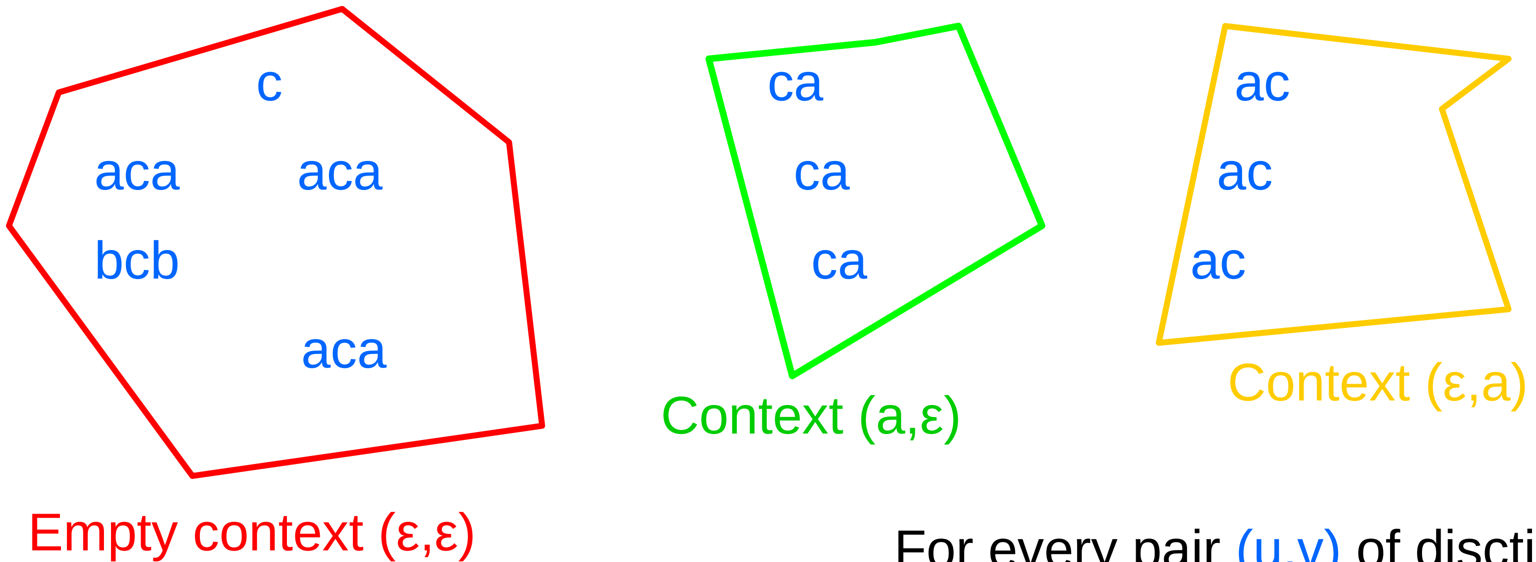
Running Example

- LS={c;aca;bcb;abcba;aacaa}



Running Example

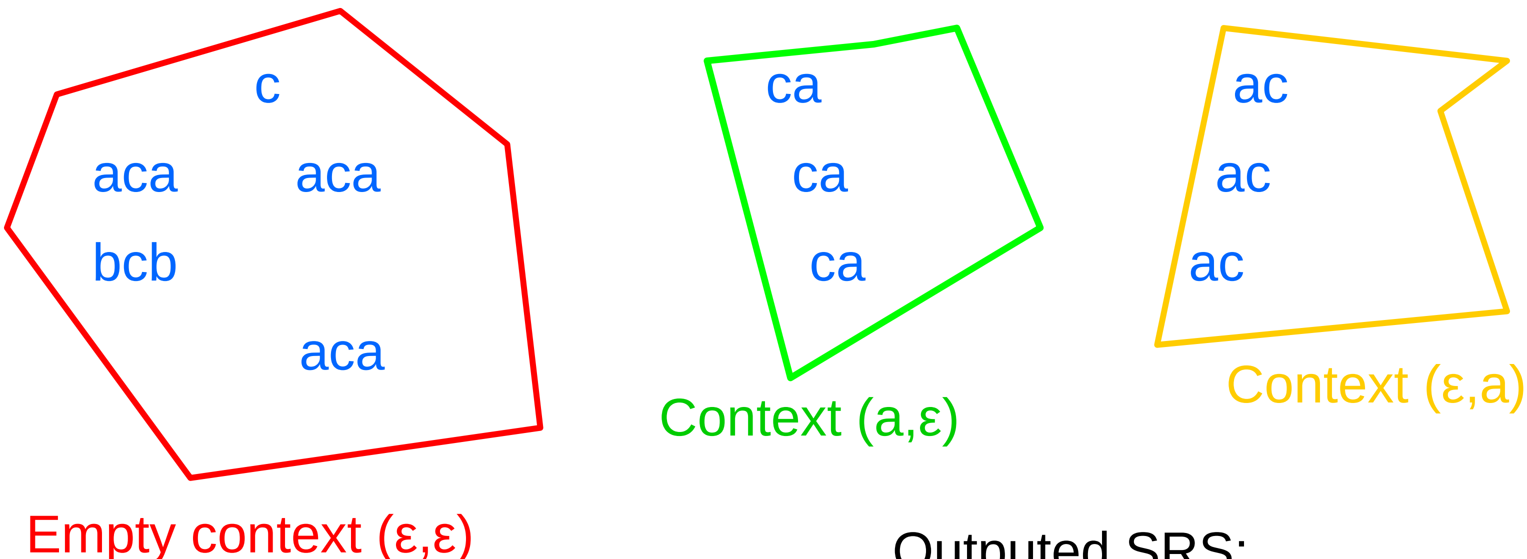
- LS={c;aca;bcb;abcba;aacaa}



For every pair (u,v) of disctinct substrings in the same component, creat a rule u → v (u > v).

Running Example

- LS={c;aca;bcb;abcba;aacaa}



Outputed SRS:

< {aca → c; bcb → c}, {c} >

Learning in the Primal

Let $D := K := F := \emptyset$;

For $n = 1,2,3,\ldots$

  let $D := \{u_1, u_2, \ldots, u_n\}$

  If $D \not\subseteq L(\mathrm{G}(K,F))$ then

    let $K := \mathrm{Sub}(D)$

  End if

  let $F := \mathrm{Con}(D)$

  output $\mathrm{G}(K,F)$

End for

Finite Kernel Property (primal)

- A CFG $G = < \Sigma, N, S, P >$ has the (one) Finite Kernel Property iff every $A \in N$ admits a characterizing string u such that $C_L(u)=\{(l,r):\exists v, N\rightarrow^* v, lvr\in L(G)\}$
- Examples: all regular languages, parenthesis languages, ...
- With enough data, one line of the observation table corresponds exactly to each non-terminal of the target grammar.
  - Adding new lines add new non-terminals (thus new rules)
  - Adding new columns remove incorrect rules