HW 2: Factors and Constraints defined with Propositional Logic

1. Locally Testable Constraints (PROP($\lhd$))
   (a) For each φ, write two words whose models satisfy it. (Assume $\Sigma = \{a, b, c\}$ and $\mathcal{M} = \langle D; \lhd, a, b, c\rangle$.)
      i. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa}$?
      ii. $\varphi \stackrel{\text{def}}{=} \neg\mathcal{M}_{aa}$?
      iii. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \vee \mathcal{M}_{ab}$?
      iv. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Rightarrow \mathcal{M}_{ab}$?
      v. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Leftrightarrow \mathcal{M}_{ab}$?
   (b) Write a constraint that says "All words with a NT sequence also have a CC sequence" (You may assume a model with features for nasals, consonants, etc.)
   (c) Prove that the constraint which says "Words have at least two *a*s" is not LT.
   (d) You are born on planet Locally 2-Testable, where everyone's DNA has a UG programmed for LT-2 constraints. You observe the word *aab*.
      i. Can you infer that *aaab* is a word in your language?
      ii. What about *ab*?

2. Piecewise Testable Constraints (PROP($<$))
   (a) For each φ, write two words whose models satisfy it. (Assume $\Sigma = \{a, b, c\}$ and $\mathcal{M} = \langle D; <, a, b, c\rangle$.)
      i. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa}$?
      ii. $\varphi \stackrel{\text{def}}{=} \neg\mathcal{M}_{aa}$?
      iii. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \vee \mathcal{M}_{ab}$?
      iv. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Rightarrow \mathcal{M}_{ab}$?
      v. $\varphi \stackrel{\text{def}}{=} \mathcal{M}_{aa} \Leftrightarrow \mathcal{M}_{ab}$?
   (b) You are born on planet Locally 2-Piecewise, where everyone's DNA has a UG programmed for PT-2 constraints. You observe the word *aabcb*.
      i. Can you infer that *aaabcb* is a word in your language?
      ii. What about *ab*? *aabbcb*? *ac*? *ca*?

3. Strictly Local Constraints (CNL($\lhd$))
   (a) Consider a Strictly 2-Local stringset L which contains the words *aa* and *ab*. Using Suffix Substitution Closure, explain what other words must be in L.
   (b) Consider the constraint *s$\ldots\int$. Show this is not SL_k for any k.
   (c) You are born on planet Strictly 2-Local, where everyone's DNA has a UG programmed for SL-2 constraints. You observe the word *aab*.
      i. Can you infer that *aaab* is a word in your language?
      ii. What about *ab*?
      iii. Do inhabitants of planet Locally 2-Testable generalize in the same way (see above)?

4. Strictly Piecewise Constraints (CNL($<$))

   (a) Consider a Strictly 2-Piecewise stringset L which contains the words *aa* and *ab*. Using subsequence closure, explain what other words must be in L.

   (b) Consider the constraint *nt. Show this is not SP_k for any k.

   (c) You are born on planet Strictly 2-Piecewise, where everyone's DNA has a UG programmed for SP-2 constraints. You observe the word *aabcb*.

      i. Can you infer that *aaabcb* is a word in your language?

      ii. What about *ab*? *aabbcb*? *ac*? *ca*?

      iii. Do inhabitants of planet Piecewise 2-Testable generalize in the same way (see above)?