Logical Representations of Syllable Structures

Chapter 4 Outline

- Representations
  - Dot
  - Flat
  - Tree
- Transformations
  - L-interpretability
  - Flat-to-Tree
  - Tree-to-Flat
  - Flat-to-Dot
  - Dot-to-Flat

Dot Structure

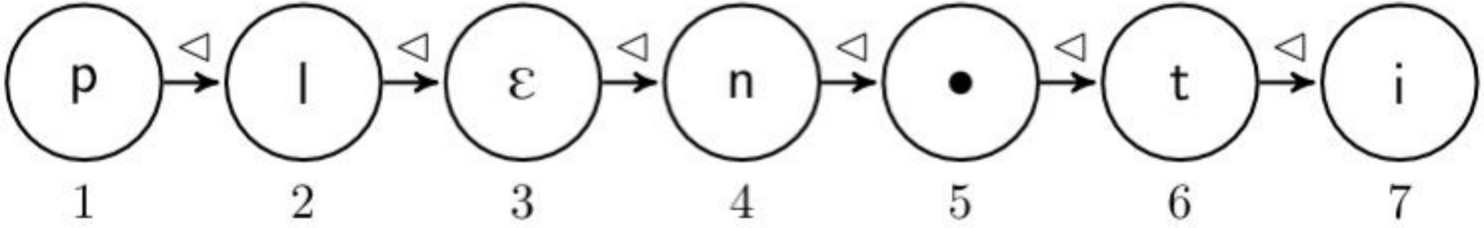
- Representation includes **string** of segments and syllable **boundaries**

$$\Sigma^{dot} \stackrel{\text{def}}{=} \mathcal{F} \cup \{\bullet\}$$

$$\mathcal{R}^{dot} \stackrel{\text{def}}{=} \{R_s \mid s \in \Sigma^{dot}\}$$

$$\mathcal{M}^{dot}_{plenty} \stackrel{\text{def}}{=} \langle \mathcal{D}; \mathcal{R}^{dot}; \{pred(x), succ(x)\} \rangle$$

Figure 4.4: $\mathcal{M}^{dot}_{plenty}$



Flat Structure

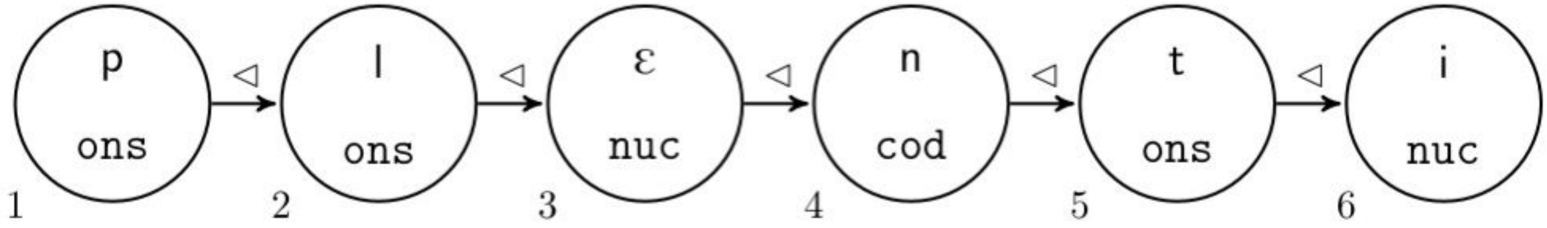
- Syllable information encoded at positions - [p] explicitly labeled as an onset, [n] explicitly labeled as a coda, etc.

$$\Sigma^{flat} \stackrel{\text{def}}{=} \mathcal{F} \cup \{\texttt{ons}, \texttt{nuc}, \texttt{cod}\}$$

$$\mathcal{R}^{flat} \stackrel{\text{def}}{=} \{R_s \mid s \in \Sigma^{flat}\}$$

$$\mathcal{M}^{flat}_{plenty} \stackrel{\text{def}}{=} \langle \mathcal{D}; \mathcal{R}^{flat}; \{pred(x), succ(x)\} \rangle$$

Figure 4.5: $\mathcal{M}^{flat}_{plenty}$



Tree Structure

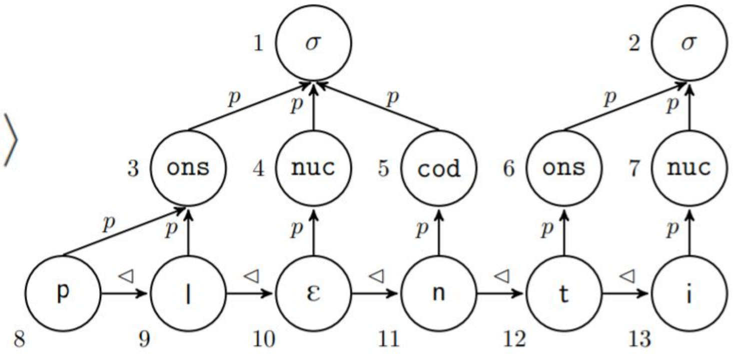
- Hierarchical structure encoding onset, nucleus, coda positions, as well as σ for a single syllable
- Inclusion of par(x) function to denote a parent node

$$\Sigma^{tree} \stackrel{\text{def}}{=} \mathcal{F} \cup \{\sigma, \mathtt{ons}, \mathtt{nuc}, \mathtt{cod}\}$$

$$\mathcal{R}^{tree} \stackrel{\text{def}}{=} \{R_s \mid s \in \Sigma^{tree}\}$$

$$\mathcal{M}^{tree}_{plenty} \stackrel{\text{def}}{=} \langle \mathcal{D}; \mathcal{R}^{tree}; \{pred(x), succ(x), par(x)\}\rangle$$

Figure 4.6: $\mathcal{M}^{tree}_{plenty}$



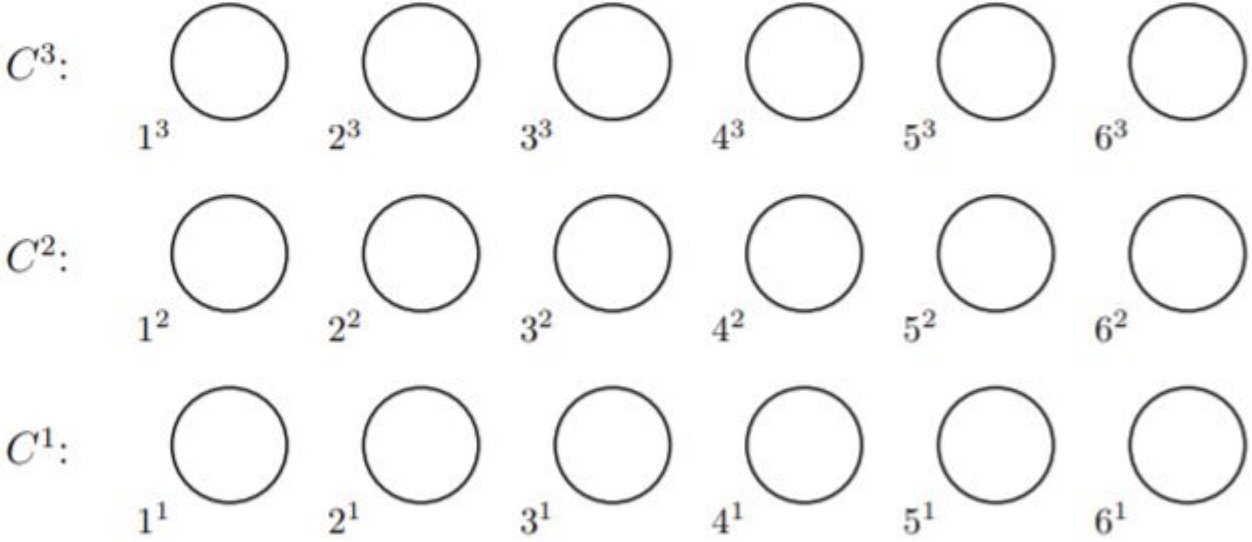
L-interpretability

- Existence of a logic L that allows for a transduction from M_1 to M_2
- The models listed here are QF-bi-interpretable (with bound on syllable size)

- Flat-to-Tree
- Tree-to-Flat
- Flat-to-Dot
- Dot-to-Flat
- **Tree-to-Dot? Dot-to-Tree?**

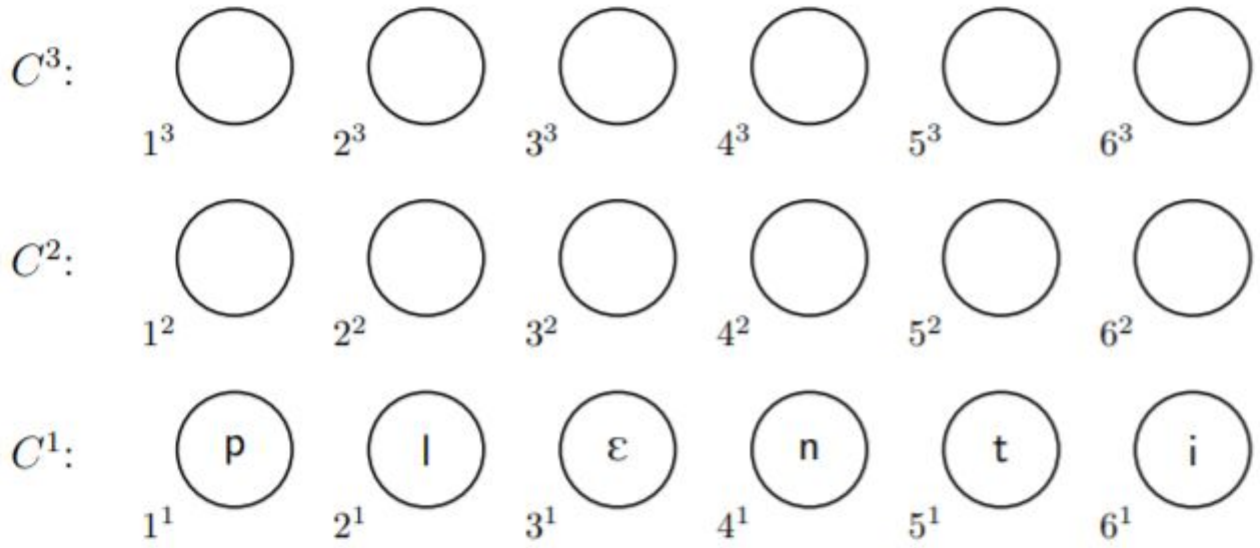
Flat-to-Tree

Figure 4.7: The codomain for $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$



Flat-to-Tree

Figure 4.8: Labels for Copy Set 1 in $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$



Flat-to-Tree

Figure 4.10: Labels for Copy Sets 1 and 2 in $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$

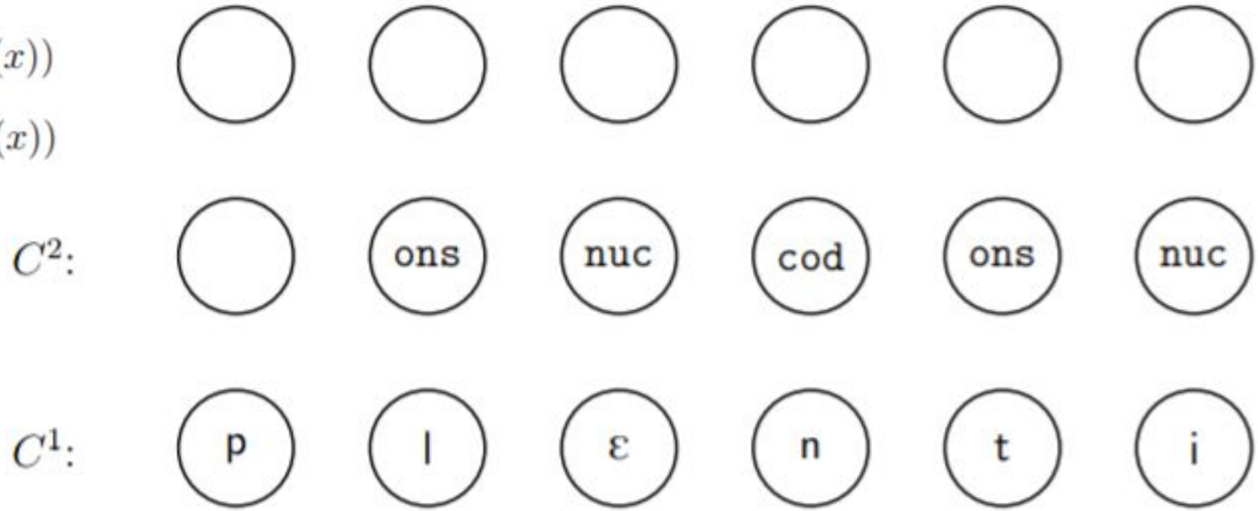
$\mathsf{ons\_1}(x) \stackrel{\text{def}}{=} \mathsf{ons}(x) \wedge \mathsf{nuc}(succ(x))$

$\mathsf{cod\_1}(x) \stackrel{\text{def}}{=} \mathsf{cod}(x) \wedge \mathsf{nuc}(pred(x))$

$\mathsf{nuc}(x^2) \stackrel{\text{def}}{=} \mathsf{nuc}(x)$

$\mathsf{ons}(x^2) \stackrel{\text{def}}{=} \mathsf{ons\_1}(x)$

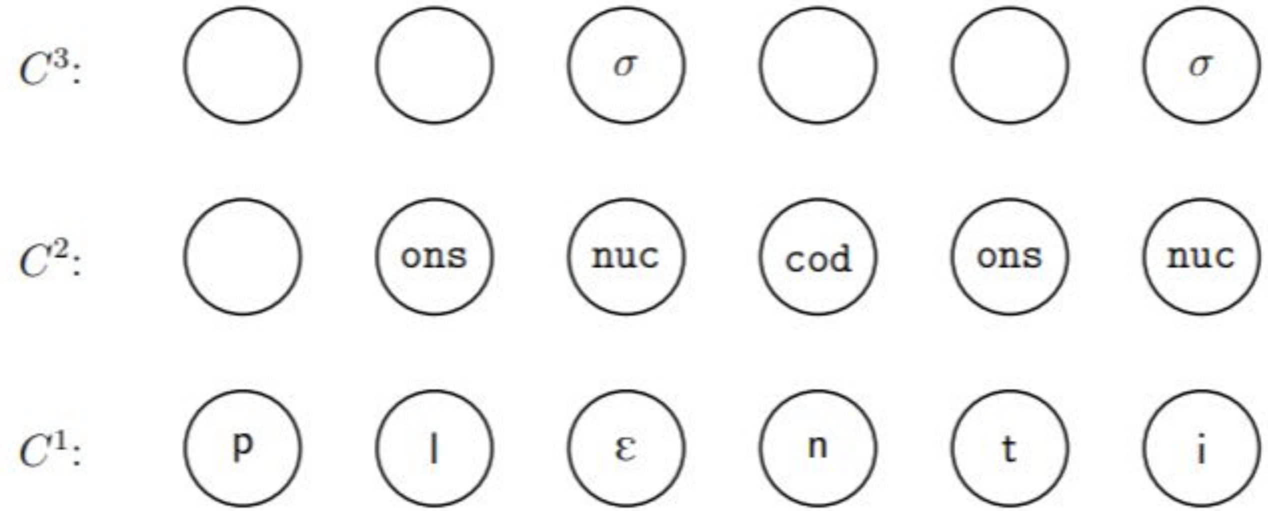
$\mathsf{cod}(x^2) \stackrel{\text{def}}{=} \mathsf{cod\_1}(x)$



Flat-to-Tree

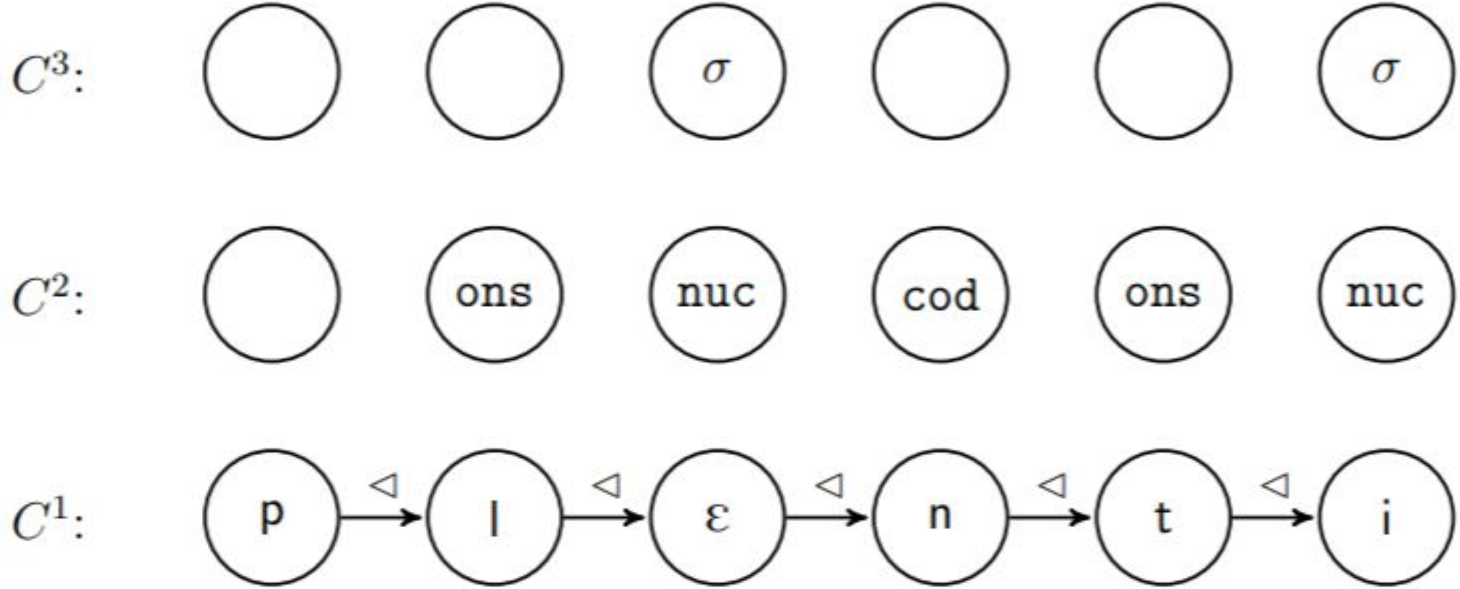
$$\sigma(x^3) \stackrel{\text{def}}{=} \mathbf{nuc}(x)$$

Figure 4.11: Labels for all three copy sets in $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$



Flat-to-Tree

Figure 4.12: The successor function in $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$

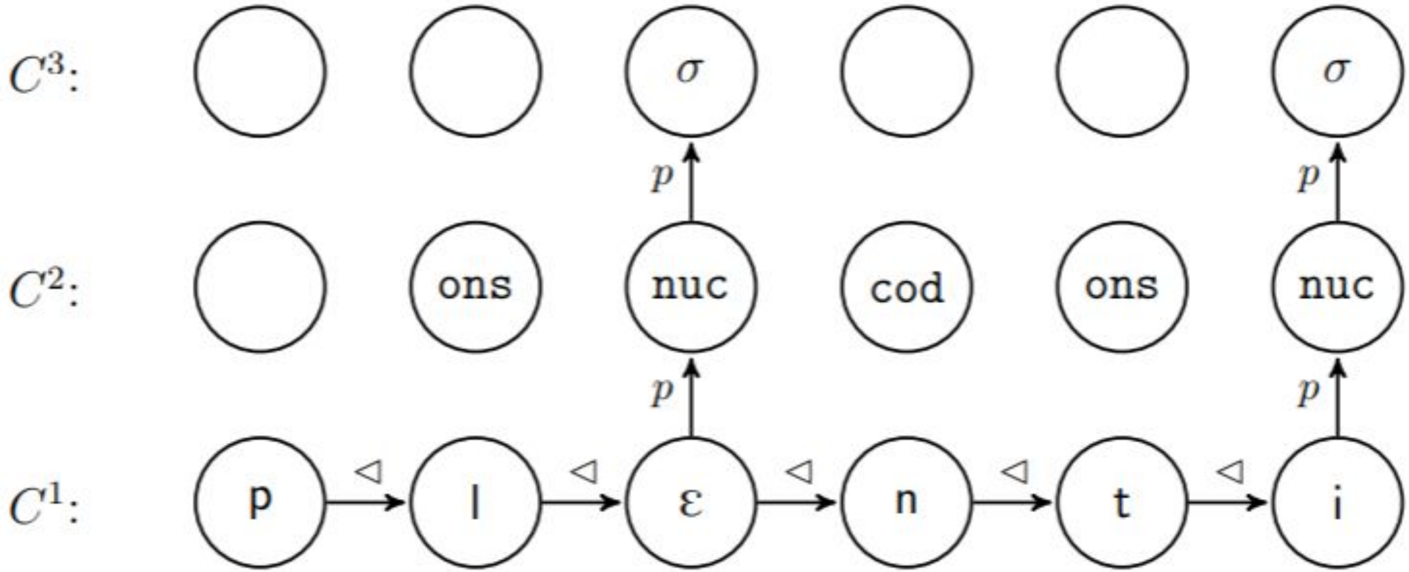


Flat-to-Tree

$$\textbf{nuc}(x) \Rightarrow par(x^2) = x^3$$

$$\textbf{nuc}(x) \Rightarrow par(x^1) = x^2$$

Figure 4.13: Some dominance information in $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$

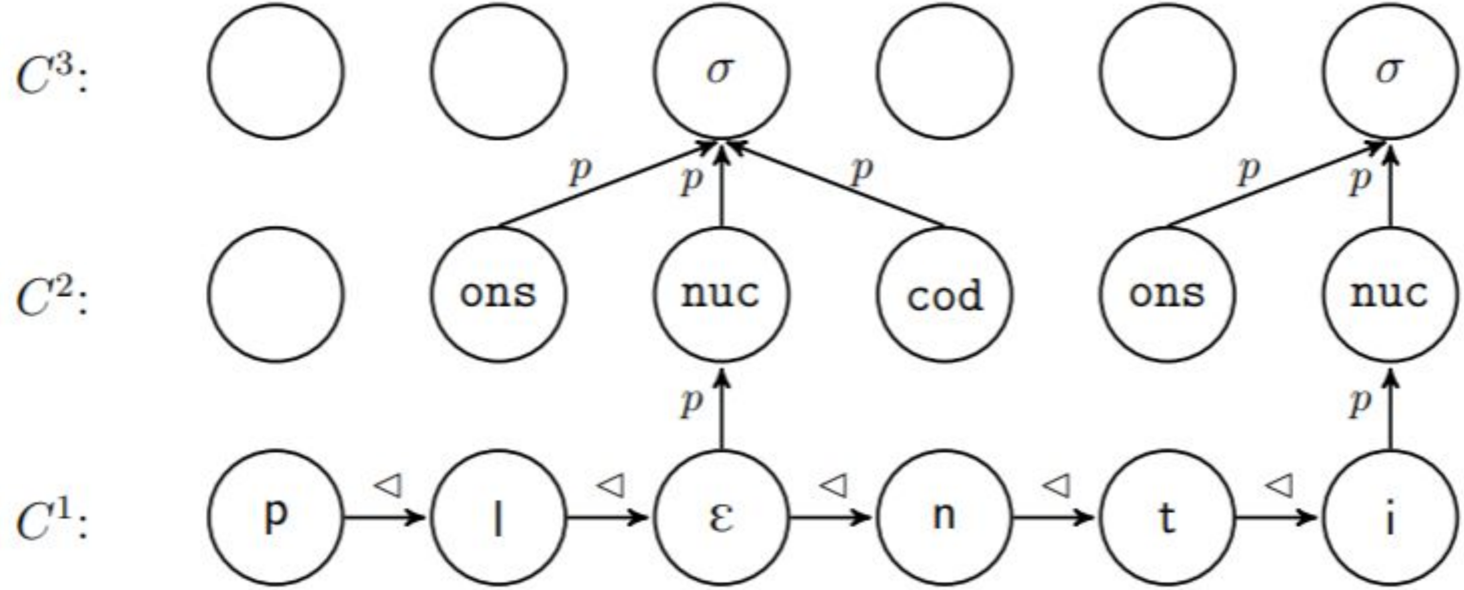


Flat-to-Tree

$$\text{ons\_1}(x) \Rightarrow par(x^2) = (succ(x))^3$$

$$\text{cod\_1}(x) \Rightarrow par(x^2) = (pred(x))^3$$

Figure 4.14: Additional dominance information in $\Gamma_{ft}(\mathcal{M}^{flat}_{plenty})$



Flat-to-Tree

$$\mathsf{ons\_}i(x) \overset{\text{def}}{=} \mathsf{ons}(x) \wedge \mathsf{ons\_}(i-1)(succ(x)) \text{ for } i \in \{2, \ldots, n\}$$

"Position x is 'onset-i' (i positions before the nucleus) iff x is labeled ons and its successor is 'onset-$(i-1)$', for i ranging from 2 to n."

$$\mathsf{cod\_}i(x) \overset{\text{def}}{=} \mathsf{cod}(x) \wedge \mathsf{cod\_}(i-1)(pred(x)) \text{ for } i \in \{2, \ldots, m\}$$

"Position x is 'coda-i' (i positions after the nucleus) iff x is labeled cod and its predecessor is 'coda-$(i-1)$', for i ranging from 2 to m."

Tree-to-Flat

Figure 4.6: $\mathcal{M}^{tree}_{plenty}$

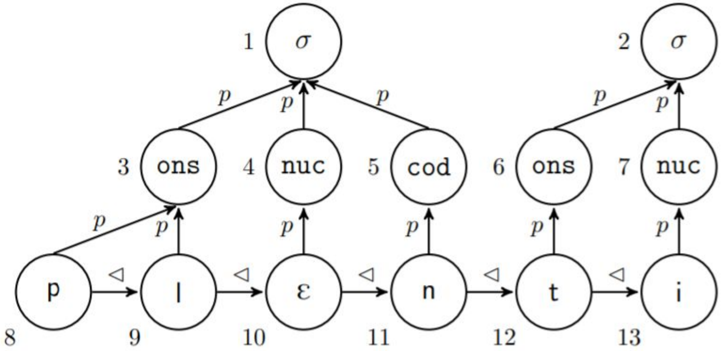
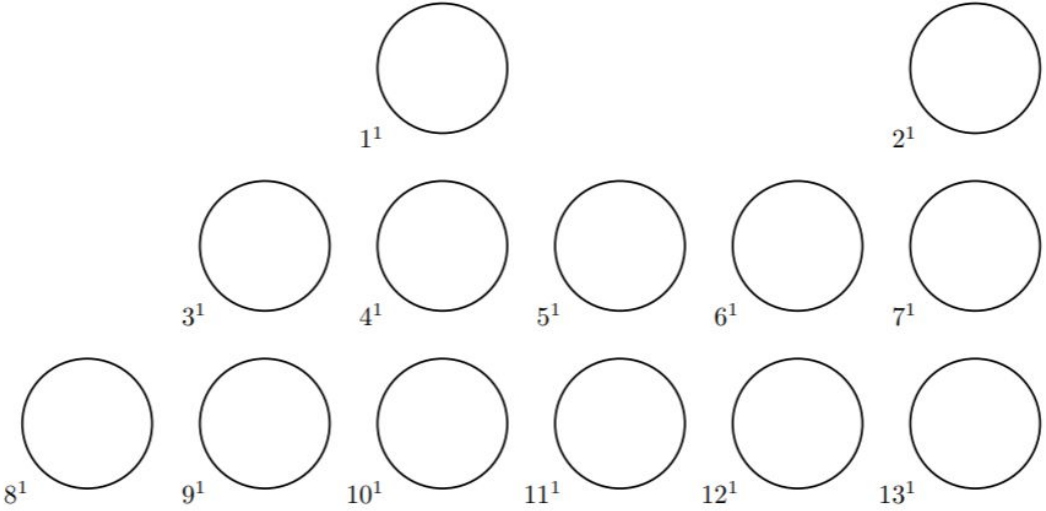


Figure 4.18: The codomain for $\Gamma_{tf}(\mathcal{M}^{tree}_{plenty})$



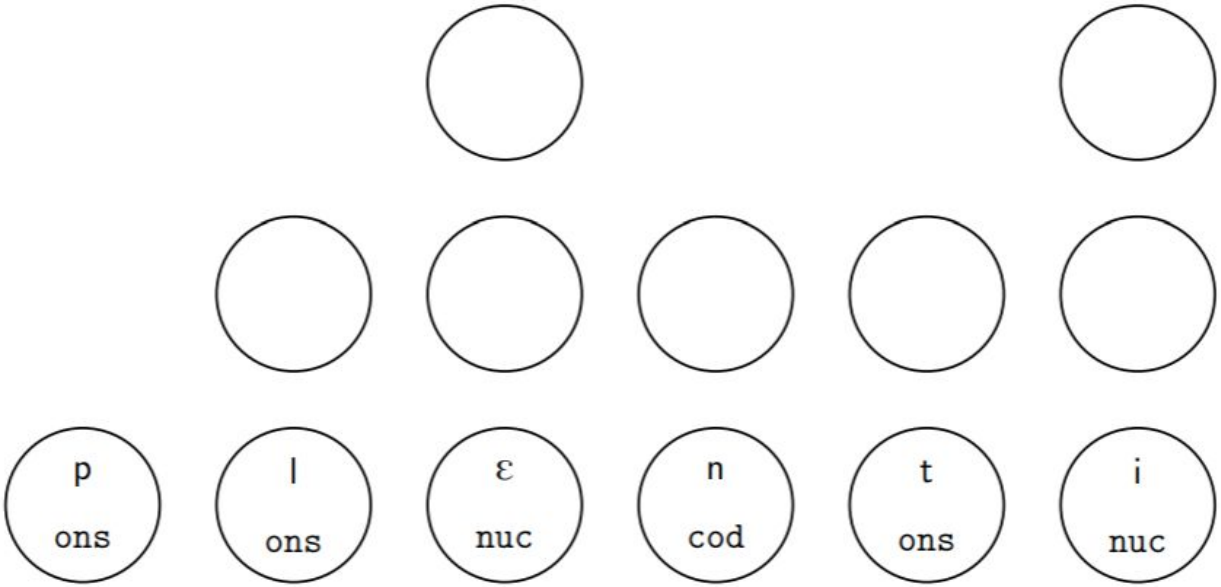
Tree-to-Flat

$$\mathtt{ons}(x^1) \overset{\text{def}}{=} \mathtt{ons}(par(x))$$

$$\mathtt{nuc}(x^1) \overset{\text{def}}{=} \mathtt{nuc}(par(x))$$

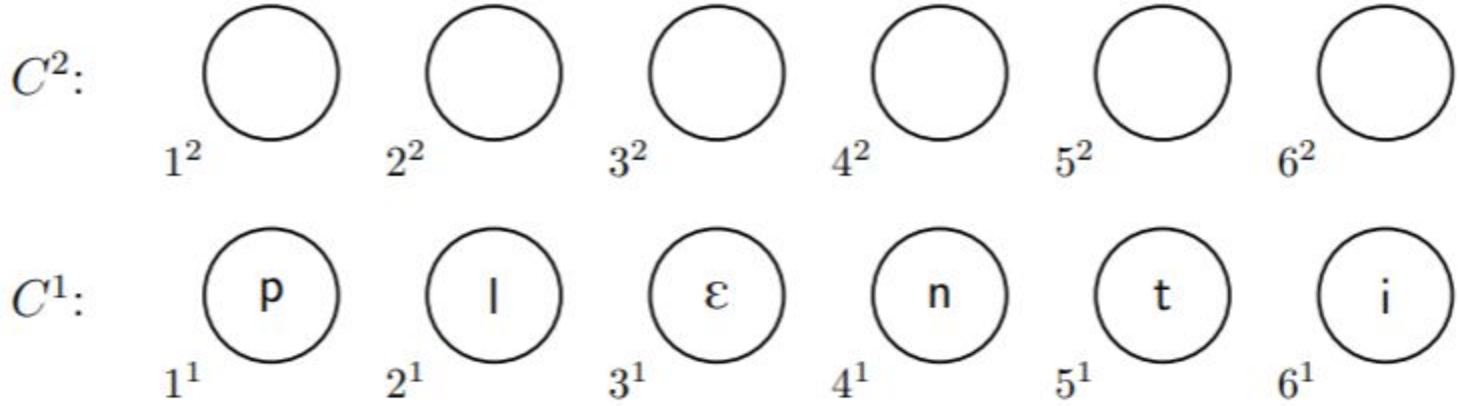
$$\mathtt{cod}(x^1) \overset{\text{def}}{=} \mathtt{cod}(par(x))$$

Figure 4.19: Unary relations for $\Gamma_{tf}(\mathcal{M}^{tree}_{plenty})$



Flat-to-Dot

Figure 4.23: Labeling Copy Set 1 in $\Gamma_{fd}(\mathcal{M}^{flat}_{plenty})$



Flat-to-Dot

$$\mathsf{c.o}(x) \stackrel{\text{def}}{=} \mathsf{cod}(x) \wedge \mathsf{ons}(succ(x))$$

$$\mathsf{n.o}(x) \stackrel{\text{def}}{=} \mathsf{nuc}(x) \wedge \mathsf{ons}((succ(x))$$

$$\mathsf{c.n}(x) \stackrel{\text{def}}{=} \mathsf{cod}(x) \wedge \mathsf{nuc}((succ(x))$$

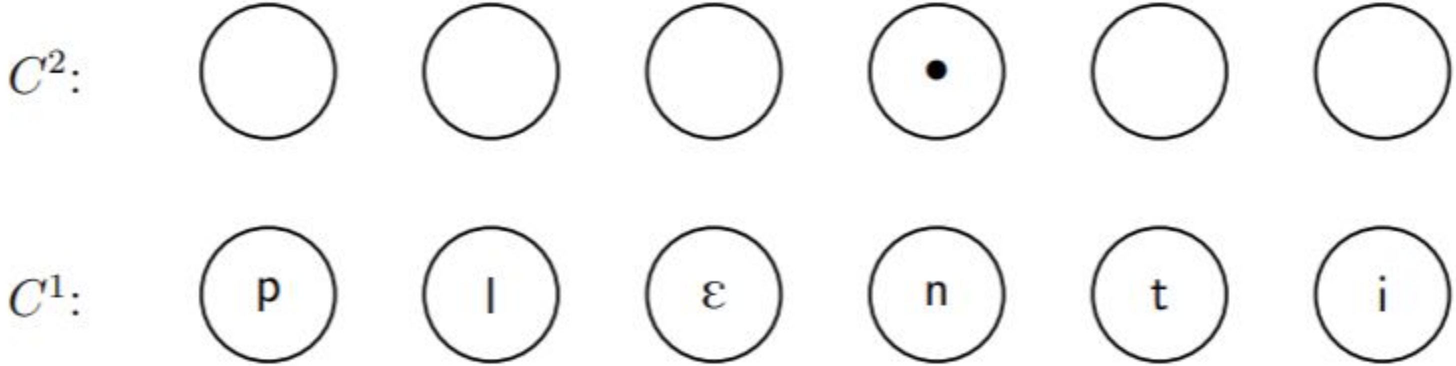
$$\mathsf{n.n}(x) \stackrel{\text{def}}{=} \mathsf{nuc}(x) \wedge \mathsf{nuc}((succ(x))$$

$$\mathsf{pre\_bound}(x) \stackrel{\text{def}}{=} \mathsf{c.o}(x) \vee \mathsf{n.o}(x) \vee \mathsf{c.n}(x) \vee \mathsf{n.n}(x)$$

$$\bullet(x^2) \stackrel{\text{def}}{=} \mathsf{pre\_bound}(x)$$

Flat-to-Dot

Figure 4.24: Labeling Copy Set 2 in $\Gamma_{fd}(\mathcal{M}^{flat}_{plenty})$



Flat-to-Dot

$$\textsf{pre\_bound}(x) \Rightarrow succ(x^1) = x^2$$

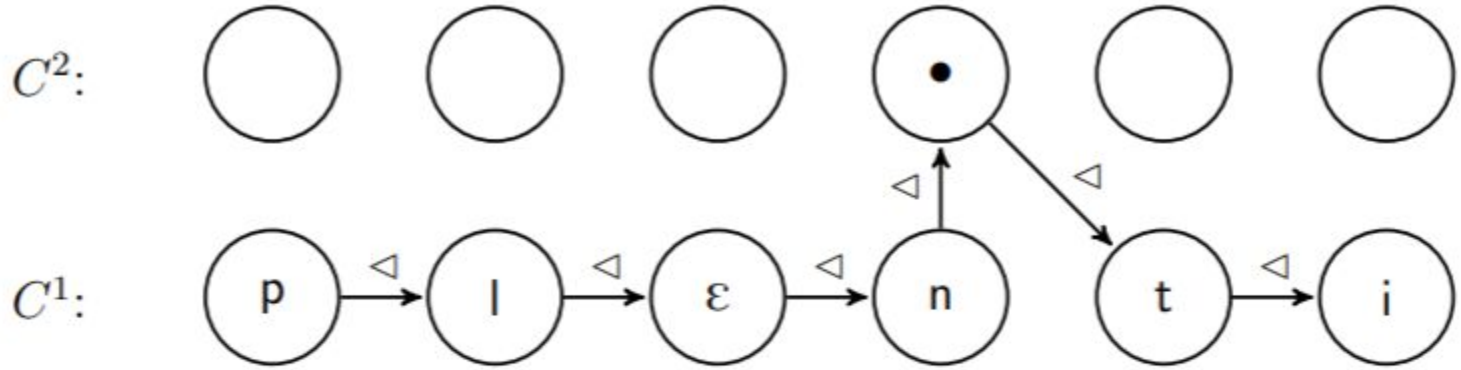
"If x is a pre-boundary position in the input, then the successor of the first copy of x is the second copy of x."

$$succ(x^2) = (succ(x))^1 \Leftrightarrow \textsf{pre\_bound}(x)$$

"The successor of the second copy of x is the first copy of the successor of x iff x is a pre-boundary position in the input."

Flat-to-Dot

Figure 4.26: All successor information in $\Gamma_{fd}(\mathcal{M}^{flat}_{plenty})$



Dot-to-Flat

Figure 4.30: Some unary relations for $\Gamma_{df}(\mathcal{M}^{dot}_{plenty})$

Dot-to-Flat

$$\texttt{nuc}(x^1) \stackrel{\text{def}}{=} \texttt{Eng\_nuc}(x)$$

$$\mathsf{ons\_1}(x) \stackrel{\text{def}}{=} \neg(\texttt{Eng\_nuc}(x) \vee \bullet(x)) \wedge \texttt{Eng\_nuc}(succ(x))$$

$$\mathsf{cod\_1}(x) \stackrel{\text{def}}{=} \neg(\texttt{Eng\_nuc}(x) \vee \bullet(x)) \wedge \texttt{Eng\_nuc}(pred(x))$$

$$\mathsf{ons\_}i(x) \stackrel{\text{def}}{=} \neg(\texttt{Eng\_nuc}(x) \vee \bullet(x)) \wedge \mathsf{ons\_}(i-1)(succ(x)) \text{ for } i \in \{2,\ldots,n\}$$

"Position x is 'onset i' iff x is not labeled Eng_nuc or $\bullet$, and its successor is 'onset $(i-1)$', for i ranging from 2 to n."

$$\mathsf{cod\_}i(x) \stackrel{\text{def}}{=} \neg(\texttt{Eng\_nuc}(x) \vee \bullet(x)) \wedge \mathsf{cod\_}(i-1)(pred(x)) \text{ for } i \in \{2,\ldots,m\}$$

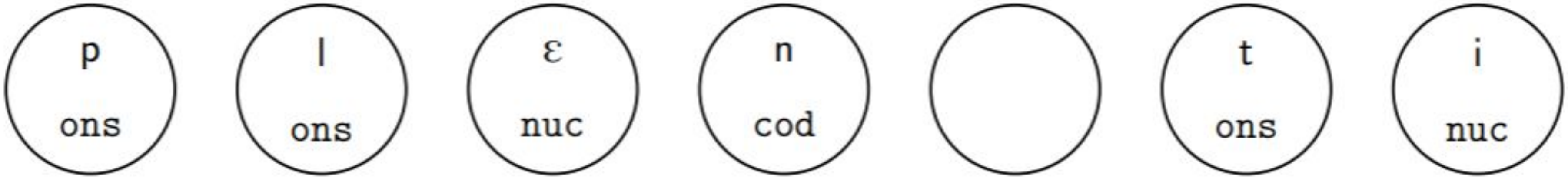
"Position x is 'coda i' iff x is not labeled Eng_nuc or $\bullet$, and its predecessor is 'coda $(i-1)$', for i ranging from 2 to m."

Dot-to-Flat

$$\mathsf{ons}(x^1) \overset{\text{def}}{=} \mathsf{ons\_}n(x) \vee \mathsf{ons\_}(n-1)(x) \vee \ldots \vee \mathsf{ons\_1}(x)$$

"Position x in Copy Set 1 is labeled `ons` iff x belongs to a contiguous string of segments (up to length n) in the input that are not labeled `Eng_nuc` or •, ending with the nucleus-adjacent onset ($\mathsf{ons\_1}$)."

Figure 4.32: Additional unary relations for $\Gamma_{df}(\mathcal{M}^{dot}_{plenty})$



Dot-to-Flat

$$succ(x^1) \stackrel{\text{def}}{=} \begin{cases} (succ(succ(x)))^1 & \Leftrightarrow \mathsf{pre\_dot}(x) \\ (succ(x))^1 & \Leftrightarrow \neg\mathsf{pre\_dot}(x) \end{cases}$$

$$pred(x^1) \stackrel{\text{def}}{=} \begin{cases} (pred(pred(x)))^1 & \Leftrightarrow \mathsf{post\_dot}(x) \\ (pred(x))^1 & \Leftrightarrow \neg\mathsf{post\_dot}(x) \end{cases}$$

Figure 4.33: $\Gamma_{df}(\mathcal{M}^{dot}_{plenty})$ fully specified