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Perception-based Grammatical Inference for Adaptive Systems

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Abstract

We introduce a learning paradigm called sensor-*identification in the limit from positive data*, where sensor is a perception module that obfuscates the learner's input. We show state-merging algorithms for learning subclasses of regular languages can be successfully applied in this framework and demonstrate its utility for robotic planning and control.
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1. Motivation

In the present ongoing work, we introduce a learning paradigm called sensor-*identification in the limit from positive data*, where sensor is a perception module that obfuscates the learner's input. In this scenario, exact identification is eschewed for converging to a grammar which generates a language approximating the target language. Successful approximation is understood as matching up to *observation-equivalence*. Theoretical work exists which addresses other kinds of imperfect presentations, oracles, and the kinds of results obtainable with them (Angluin and Laird, 1988; Stephan, 1995; Fulk and Jain, 1996; Case and Jain, 2001; Tantini et al., 2006).

Our motivation primarily comes from robotics, where a robot is attempting to satisfy a behavior specification while facing dynamic, adversarial rule-governed behavior from its environment. Its sensors are limited in that it cannot observe every parameter of the environment at each moment. If it could then grammatical inference can be used in conjunction with standard game theoretic analysis to identify the environment and plan accordingly (Chandlee et al., 2012; Fu et al., 2013, 2014a). However, the game-theoretic literature (Arnold et al., 2003; Chatterjee et al., 2006) shows that it is possible to synthesize correct controllers (i.e. find winning strategies) even for games with *imperfect* information (where players only have partial information about the state of the game). The techniques in Chandlee et al. (2012); Fu et al. (2013) and Fu et al. (2014a) allow imperfect games to be constructed from imperfect, but consistent, models of the environment. What is missing then is a way to identify these models from imperfect observations.

This work fills this gap. Our basic strategy is to convert learning solutions for the identification in the limit from positive data paradigm to solutions in the paradigm below. We focus on regular classes of languages learnable with state-merging algorithms, which are well-studied (de la Higuera, 2010).

2. Main results

For any L, let $\sim_L$ be the Myhill-Nerode equivalence relation for L: so $w \sim_L w' \Leftrightarrow \{v \in \Sigma^* \mid wv \in L\} = \{v \in \Sigma^* \mid w'v \in L\}$. Given as input a finite sample $S \subset \Sigma^*$, a state-merging algorithm $\mathfrak{A}$ determines an equivalence relation $\sim_{\mathfrak{A}}$ over Σ^*. States in a prefix tree acceptor (PTA) of S which correspond to equivalent prefixes according to $\sim_{\mathfrak{A}}$ are then merged. For any L, if $\sim_{\mathfrak{A}}$ is of finite index and refines $\sim_L$ then $\mathfrak{A}$ identifies L in the limit from positive data. If $\mathfrak{A}$ does this for every $L \in \mathcal{L}$ then $\mathfrak{A}$ identifies $\mathcal{L}$ in the limit from positive data.

Sensor models have been proposed before (Cassandras and Lafortune, 2008; Luo et al., 2011; Fu et al., 2014b); the definition below subsumes them all.

Definition 1 (Sensor model) *A sensor model is* $\mathsf{sensor} = \langle \Theta, \Sigma, \sim_\theta (\forall \theta \in \Theta), L_\Theta \rangle$ *where*

- Θ *and* Σ *are finite, ordered sets of alphabets (the former being the* sensor configurations*).*
- *For all* $\theta \in \Theta$, $\sim_\theta$ *is an equivalence relation on* Σ*. If* $\sigma_1 \sim_\theta \sigma_2$ *then* σ_1 *is* indistinguishable *from* σ_2 *under sensor configuration* θ*. Let* $[\sigma]_\theta = \{\sigma' \in \Sigma \mid \sigma' \sim_\theta \sigma\}$*.*
- $L_\Theta \subseteq \Theta^*$ *is regular and represents the permissible sequences of sensor configurations.*

Define the left (π_1) and right (π_2) projections of words $w \in (\Theta \times \Sigma)^*$ recursively as follows: $\pi_1(\lambda) = \pi_2(\lambda) = \lambda$, and $\pi_1(w \cdot (\theta, \sigma)) = \pi_1(w) \cdot \theta$ and $\pi_2(w \cdot (\theta, \sigma)) = \pi_2(w) \cdot \sigma$. Letting $\hat{\Sigma} = \{[\sigma]_\theta \mid \sigma \in \Sigma, \theta \in \Theta\}$, we also define π_1 and π_2 for words $\hat{w} \in (\Theta \times \hat{\Sigma})^*$ in the same way. For each $w \in (\Theta \times \Sigma)^*$, the *possible observations* of w are defined recursively with $\mathsf{obs} : (\Theta \times \Sigma)^* \to \hat{\Sigma}^*$ as follows. $\mathsf{obs}(\lambda) = \{\lambda\}$ and $\mathsf{obs}(w \cdot (\theta, \sigma)) = \mathsf{obs}(w) \cdot [\sigma]_\theta$. Similarly, for each $u \in \Theta^*$, a sensor model recursively induces an equivalence relation $\sim_u$ over Σ^*: $\lambda \sim_\lambda \lambda$, and $(\forall \sigma_1, \sigma_2 \in \Sigma, v_1, v_2 \in \Sigma^*, \theta \in \Theta, u \in \Theta^*)\big[v_1 \sim_u v_2 \Rightarrow (v_1\sigma_1 \sim_{u\theta} v_2\sigma_2 \Leftrightarrow \sigma_1 \sim_\theta \sigma_2)\big]$. Let $[v]_u = \{v' \in \Sigma^* \mid v \sim_u v'\}$, which denotes equivalent strings in Σ^* according to $u \in \Theta^*$.

Lemma 2 *For all* $w \in (\Theta \times \Sigma)^*$*, if* $\pi_1(w) = u$ *and* $\pi_2(w) = v$ *then* $|w| = |u| = |v|$ *and* $[v]_u = \mathsf{obs}(w)$ *is a finite subset of* Σ^**.*

Definition 3 (Observation-equivalence) *For any* $L \subseteq \Sigma^*$*, let* $L^{=n} = \{w \in L \mid |w| = n\}$*. According to model* sensor*, languages* $L, L' \subseteq \Sigma^*$ *are* observation-equivalent *if*

$$(\forall v \in L)(\exists v' \in L')(\forall u \in L_\Theta^{=|v|})[v \sim_u v'] \wedge (\forall v' \in L')(\exists v \in L)(\forall u \in L_\Theta^{=|v'|})[v \sim_u v'].$$

Definition 4 (Sensor-identification in the limit) *We consider a sensor model* $\mathsf{sensor} = \langle \Theta, \Sigma, \sim_\theta (\forall \theta \in \Theta), L_\Theta \rangle$ *and family of languages* $\mathcal{L}$ *over* Σ*.*

- *For each* $L \in \mathcal{L}$*, let* $L_{\mathsf{system}} = \{w \in (\Theta \times \Sigma)^* \mid \pi_1(w) \in L_\Theta \wedge \pi_2(w) \in L\}$ *and* $L_{\mathsf{sensor}} = \{\hat{w} \in (\Theta \times \hat{\Sigma})^* \mid (\exists w \in L_{\mathsf{system}} \wedge \pi_1(\hat{w}) = \pi_1(w) \wedge \pi_2(\hat{w}) = \mathsf{obs}(w)\}$*.* L_{system} *represents the actual behavior of the system and* L_{sensor} *represents its possible behaviors consistent with the possible observations under the model* sensor*.*

- $\mathcal{L}$ *is* Sensor-identifiable in the limit from positive data *if there exists an algorithm* $\mathfrak{A}$ *such that for all* $L \in \mathcal{L}$*, for any presentation* ϕ *of* L_{sensor}*, there exists* $n \in \mathbb{N}$ *such that for all* $m \geq n$*,* $\mathfrak{A}(\phi[m]) = \mathfrak{A}(\phi[n]) = G$*, and* $L(G)$ *is observation-equivalent to* L*.*

Lemma 5 *If* L_Θ *and* L *are regular then* $\sim_{\mathsf{system}}$ *is of finite index and a right congruence. Furthermore,* $w \sim_{\mathsf{system}} w' \Leftrightarrow \pi_1(w) \sim_{L_\Theta} \pi_1(w') \wedge \pi_2(w) \sim_L \pi_2(w')$*.*

A right congruence $\sim$ over Σ^* induces a relation $\approx$ among elements of $\mathscr{P}(\Sigma^*)$: $X \approx Y \Leftrightarrow (\forall x \in X)(\exists y \in Y)(x \sim y) \wedge (\forall y \in Y)(\exists x \in X)[x \sim y]$. Since elements of $\hat{\Sigma}^*$ can be understood as subsets of Σ^*, $\approx_L$ is meaningful on $\hat{\Sigma}^*$.

Lemma 6 *If* $\sim_{\mathsf{system}}$ *is of finite index and a right congruence then so is* $\sim_{\mathsf{sensor}}$*. Furthermore,* $w \sim_{\mathsf{sensor}} w' \Leftrightarrow \pi_1(w) \sim_{L_\Theta} \pi_1(w') \wedge \pi_2(w) \approx_L \pi_2(w')$*.*

By Lemmas 5 and 6, there is a DFA A accepting L_{sensor}. A defines a class of languages $\mathcal{L}_{\mathsf{sensor}}$ over Σ. Each $L \in \mathcal{L}_{\mathsf{sensor}}$ is obtained by replacing each label (which is an element of $\Theta \times \hat{\Sigma}$) of each transition in A with one element drawn from the label's right projection (thus the drawn element belongs to Σ). These choices can be made consistent since Σ is ordered.

Lemma 7 *Any* $L' \in \mathcal{L}_{\mathsf{sensor}}$ *is observation equivalent to* L*.*

Theorem 8 (Main result) *Let* $\mathcal{L}$ *be identifiable in the limit from positive data by a state-merging algorithm* $\mathfrak{A}$ *and consider* $\mathsf{sensor} = \langle \Theta, \Sigma, \sim_\theta \ (\forall \theta \in \Theta), L_\Theta \rangle$*. There exists an algorithm* $\mathfrak{B}$ *which* Sensor*-identifies* $\mathcal{L}$ *in the limit from positive data.*

Proof. The state-merging algorithm $\mathfrak{A}$ which identifies $\mathcal{L}$, the equivalence relations $\theta \in \Theta$ on Σ, and L_Θ define another algorithm $\mathfrak{B}$ which takes as input a finite set $S \subset L_{\mathsf{sensor}}$. $\mathfrak{B}$ builds a PTA for S and merges prefixes according to $\sim_{\mathfrak{B}}$, defined as follows:

$$\hat{w} \sim_{\mathfrak{B}} \hat{w}' \Leftrightarrow \pi_1(\hat{w}) \sim_{L_\Theta} \pi_1(\hat{w}') \wedge \pi_2(\hat{w}) \approx_{\mathfrak{A}} \pi_2(\hat{w}').$$

Since L_Θ is regular, we assume it is given in terms of its minimal DFA and so $\sim_{L_\Theta}$ can be computed. Also, $\approx_{\mathfrak{A}}$ can be computed since $\sim_{\mathfrak{A}}$ can be computed and every $\mathsf{obs}(w)$ ($w \in L_{\mathsf{system}}$) is a finite set. In the limit, $\sim_{\mathfrak{B}}$ is of finite index because $\sim_{\mathfrak{A}}$ is of finite index. Also in the limit, $\sim_{\mathfrak{B}}$ refines $\sim_{\mathsf{sensor}}$ because $\sim_{\mathfrak{A}}$ refines $\sim_L$ in the limit and by definition of $\approx$. Thus this acceptor recognizes the same language as L_{sensor}, and by Lemma 7, a language L' observation-equivalent to L can be obtained. Convergence to L' is guaranteed by drawing least elements to find it. □

We also demonstrate the effectiveness of the methods above with a robot motion planning problem in an office-like environment based on the one in Fu et al. (2013). Once a model of partially observed language for the environment is identified, a product operation of the transition system of the robot and the learned model of partially observed environment yields a game with imperfect information, on which existing tools from algorithmic game theory can be applied to construct an observation-based strategy. We refer readers to Chandlee et al. (2012) for details on the construction of this game, and to Chatterjee et al. (2006) for the synthesis algorithm with partial information.

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