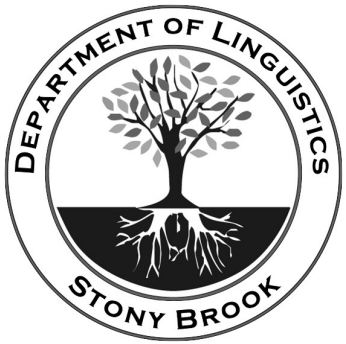
Types of Orderings and Their Implications for Phonology and Morphology

Jeffrey Heinz



Stony Brook University

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2025 05 10

Today

- Informally, by *ordering* I mean a systematic way in which the ordering of the elements in the input are related to the ordering of the elements in the output.
- I discuss two kinds of orderings from a mathematical and computational perspective and argue that phonology is characterized by one, and morphology by the other.

Today

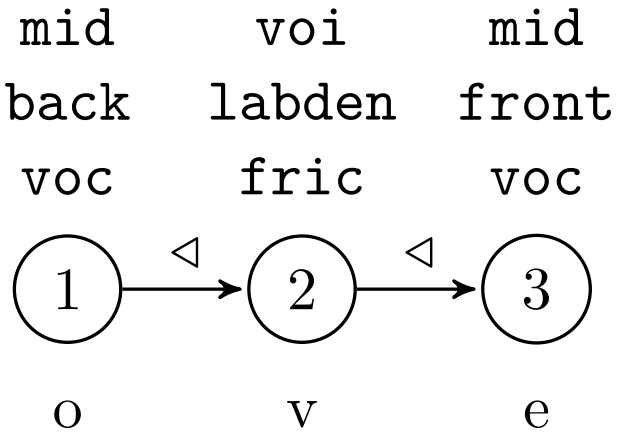
- Proposed universal properties — such as these orderings — make predictions for typology, pyscho- and neuro-linguistics, and acquisition/learnability.
- They are also stated at a more abstract level, with implications for different grammatical formalisms.

For example, "phonology is order-preserving" implies

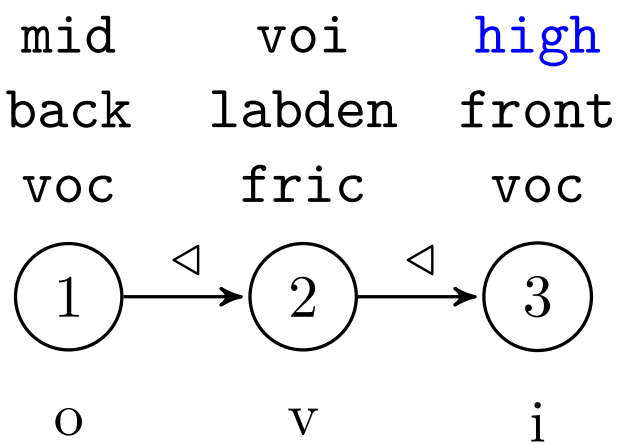
- for RBP: there are no rules like $\overset{\text{XY}}{12} \rightarrow 21$
- for OT: there is no constraint like Linearity

Mapping Inputs to Outputs

Finnish word-final /e/ raising (Odden 2014)

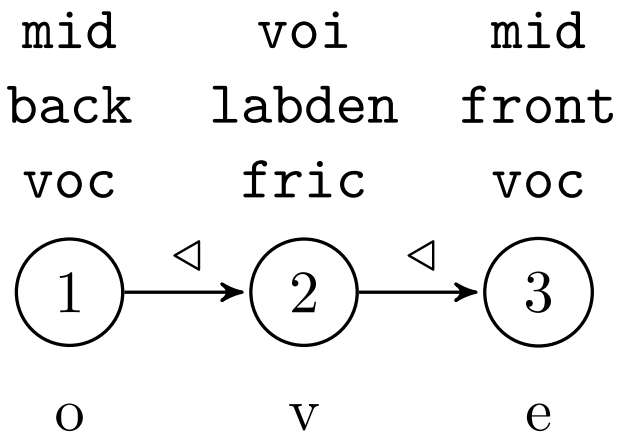


'door, nom-sg.'



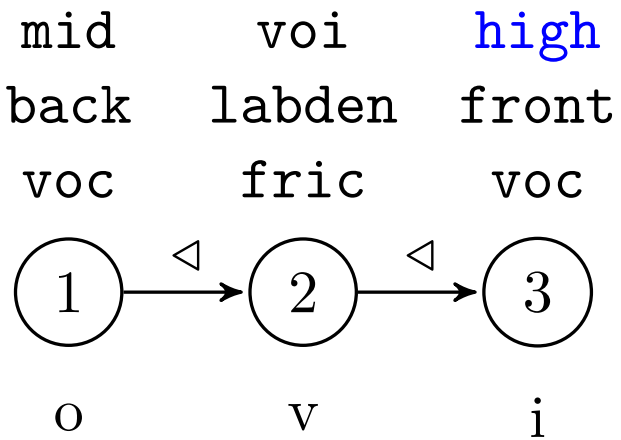
Mapping Inputs to Outputs

Finnish word-final /e/ raising (Odden 2014)



'door, nom-sg.'

- There are 3 positions.
- Positions have properties.
- Positions are ordered according to a relation.
- Input and output positions correspond.



Introducing Order Preservation with Metathesis

Uab Meto (Mooney, 2022): /kokɪse/→ [kɔ͡ɪkse] 'the bread'

Input:

k		o		k		ɪ		s		e
①	◁	②	◁	③	◁	④	◁	⑤	◁	⑥

Introducing Order Preservation with Metathesis

Uab Meto (Mooney, 2022): /kokɪse/→ [kɔ͡ɪkse] 'the bread'

Output: (transpose positions, not properties of those positions)

✗

k		o		ɪ		k		s		e
①	◁	②	◁	④	◁	③	◁	⑤	◁	⑥

Not order preserving

Introducing Order Preservation with Metathesis

Uab Meto (Mooney, 2022): /kokɪse/→ [kɔ͡ɪkse] 'the bread'

Output: (change properties, but not the order of the positions)

✓

k		o		ɪ		k		s		e
①	◁	②	◁	③	◁	④	◁	⑤	◁	⑥

Order preserving

Outputs differ in size from inputs

Lardil word-final vowel deletion:

/yalulu/ → [yalul] 'flame, uninflected'

Malagasy word-final vowel epenthesis:

/tanan/ → [tanana] 'hand'

Indonesian plural formation:

/buku/ → [bukubuku] 'book, plural'

How do the input and output positions correspond?

Kenstowicz and Kisseberth 1979, Cohn 1989, Albro 2005, O'Neill 2015

Logical Transductions

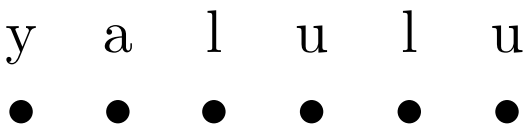
1. Logical formulas *license* output positions, and unlicensed positions – and all associated relations – delete.
2. A transduction comes with a fixed *copy size*. This specifies how many copies of each input position is licensable in the output.

Courcelle 1994, Courcelle and Engelfriedt 2012

Examples

Lardil: /yalulu/ → [yalul]. 1 copy per position.

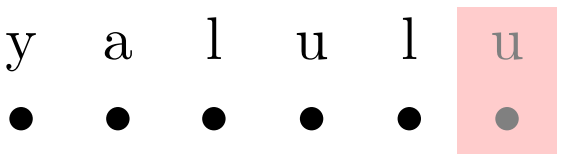
Input:



Examples

Lardil: /yalulu/ → [yalul]. 1 copy per position.

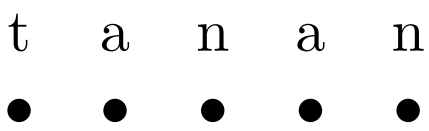
Output:



Examples

Malagasy: /tanan/ → [tanana]. 2 copies per position.

Input:



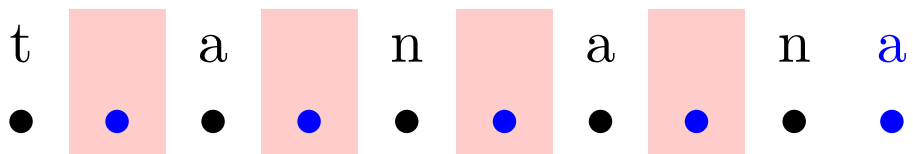
Examples

Malagasy: /tanan/ → [tanana]. 2 copies per position.

Output:

Malagasy: /tanan/ → [tanana]. 2 copies per position.

Output:



Examples

Indonesian: /buku/ → [bukubuku]. 2 copies per position.

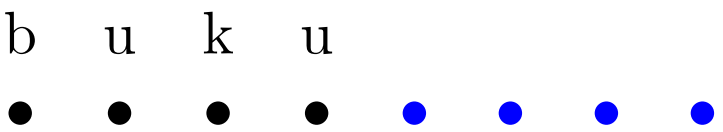
Input:

b	u	k	u
•	•	•	•

Examples

Indonesian: /buku/ → [bukubuku]. 2 copies per position.

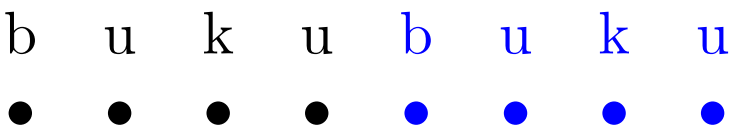
Output:



Examples

Indonesian: /buku/ → [bukubuku]. 2 copies per position.

Output:



Two natural orderings for linearizing the output

Consider 4 positions and a copyset of size 3.

		Position			
		1	2	3	4
Copy	1	●	●	●	●
	2	●	●	●	●
	3	●	●	●	●

position-wise

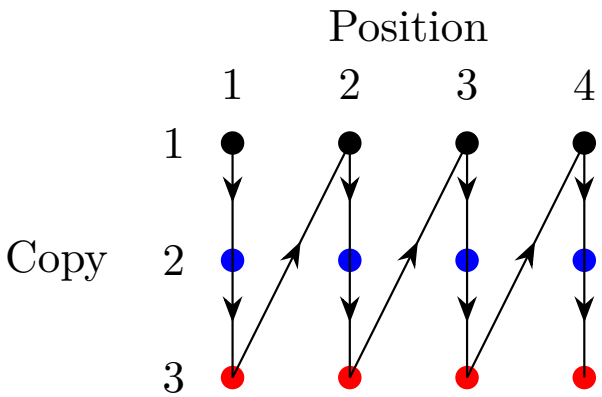
order by position, then by copy

copy-wise

order by copy, then by position

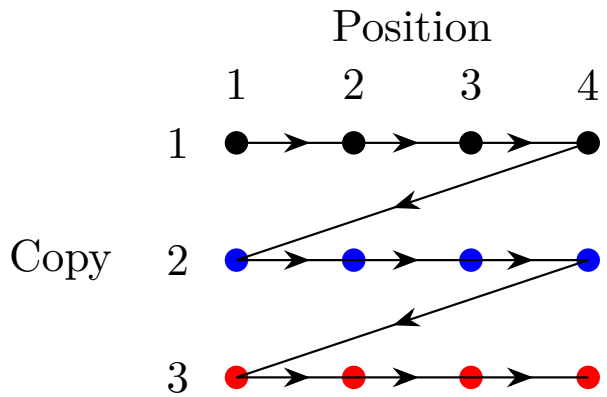
Another view

Consider 4 positions and a copyset of size 3.



position-wise

order by position, then by copy



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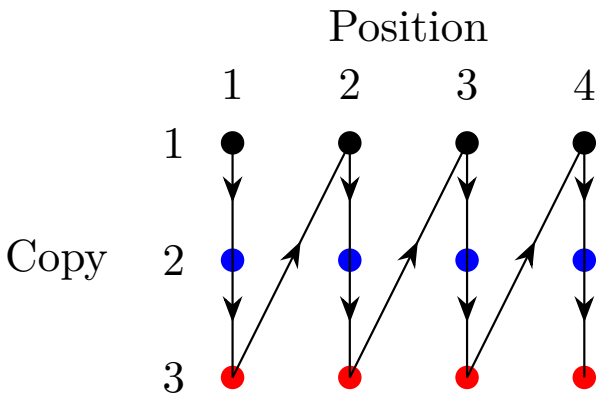
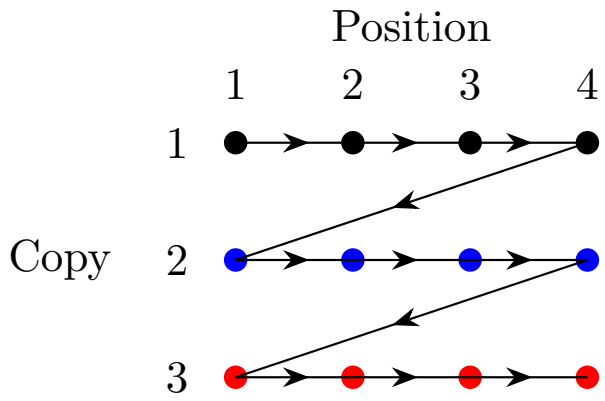
Another view

Consider 4 positions and a copyset of size 3.



position-wise

order by position, then by copy



copy-wise

order by copy, then by position

*In theoretical computer science, only the position-wise ordering is referred to as "order-preserving".

Technical Note

- Both the position-wise and copy-wise orderings are easy to define in First Order logic with general precedence: FO($<$).
- They are not First Order definable with successor, FO($\lhd$), but they are definable with Monadic Second Order logic and successor, MSO($\lhd$).

Thesis

- **Phonological** processes obey **position-wise** ordering.
- **Morphological** processes obey **copy-wise** ordering.

Many processes are both position-wise and copy-wise

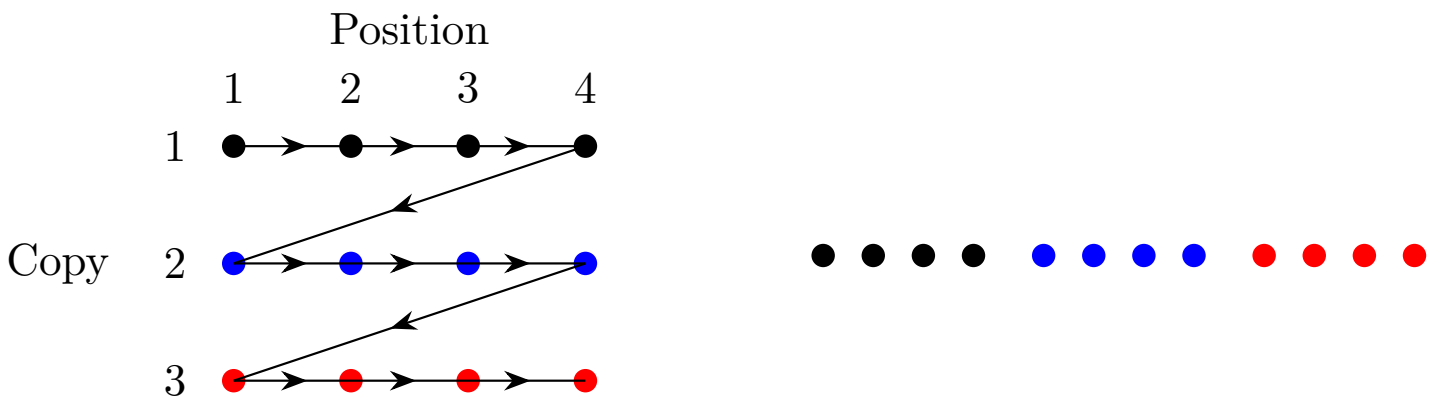
- prefxation, suffixation, affixation, infixation
- deletion
- local assimilation
- consonant harmony
- vowel harmony
- ...

Evidence from total reduplication

Indonesian plural formation:

/buku/ → [bukubuku] 'book, plural'

copy-wise: order by copy, then by position



Total reduplication is ...

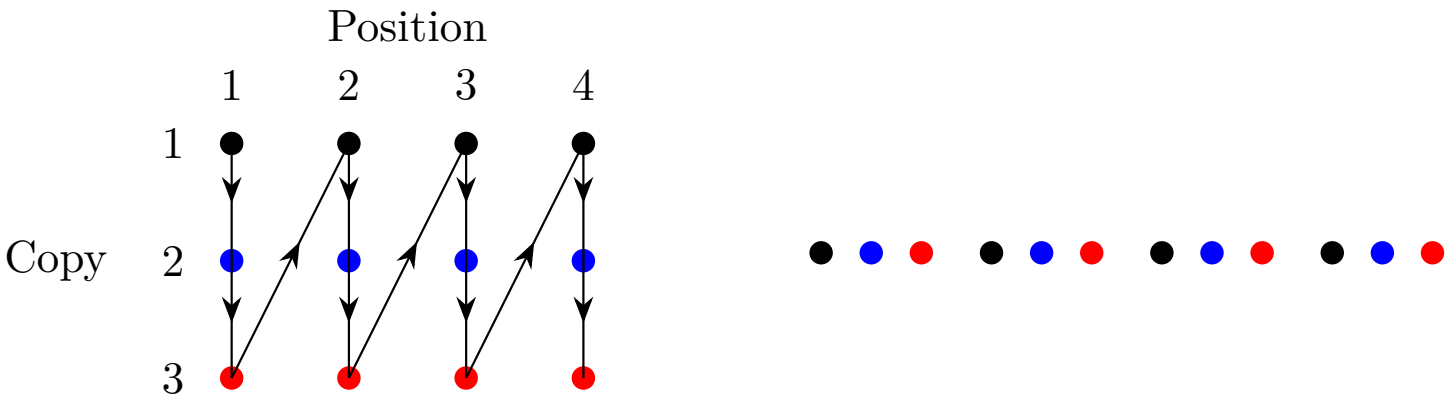
a morphological – not phonological – process, and reflects the copy-wise ordering.

Evidence from total reduplication

Indonesian plural formation:

/buku/ → [bukubuku] 'book, plural'

position-wise: order by position, then by copy



Total reduplication is ...

difficult (impossible?) to do with a position-wise ordering.

Evidence from metathesis

Uab Meto (Mooney, 2022):

/kokɪse/→ [kôɪkse] 'the bread'

Kwara'ae (Heinz, 2005; Takahashi, 2019):

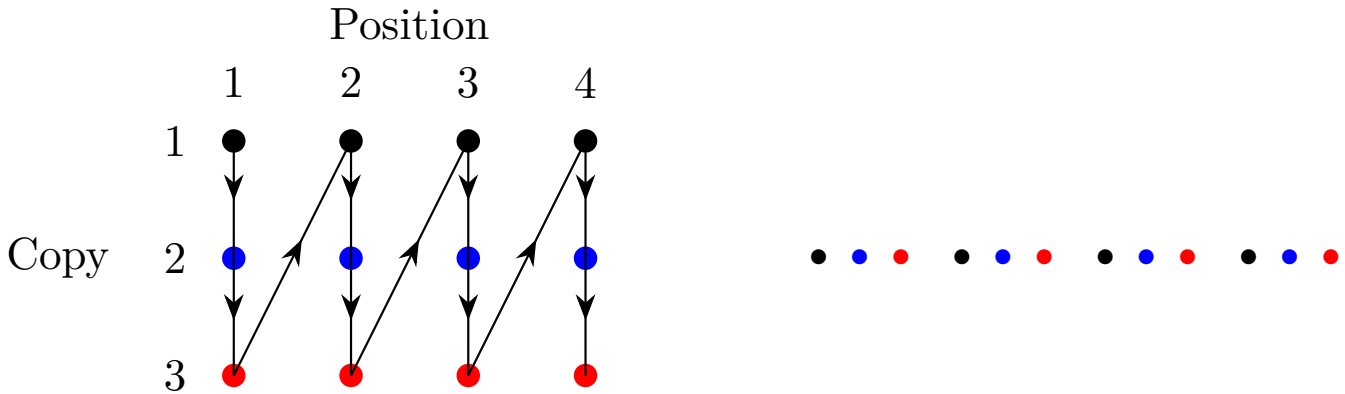
/ketalaku/ → [k̟eatla̟uk] 'my height'

Takahashi (2019) and Mooney (2022)

Both argue phonology contains no transposition operation, and that metathesis is the outcome of copy and deletion processes (cf. Blevins and Garrett 1998).

Copy and Delete, Position-wise

position-wise: order by position, then by copy



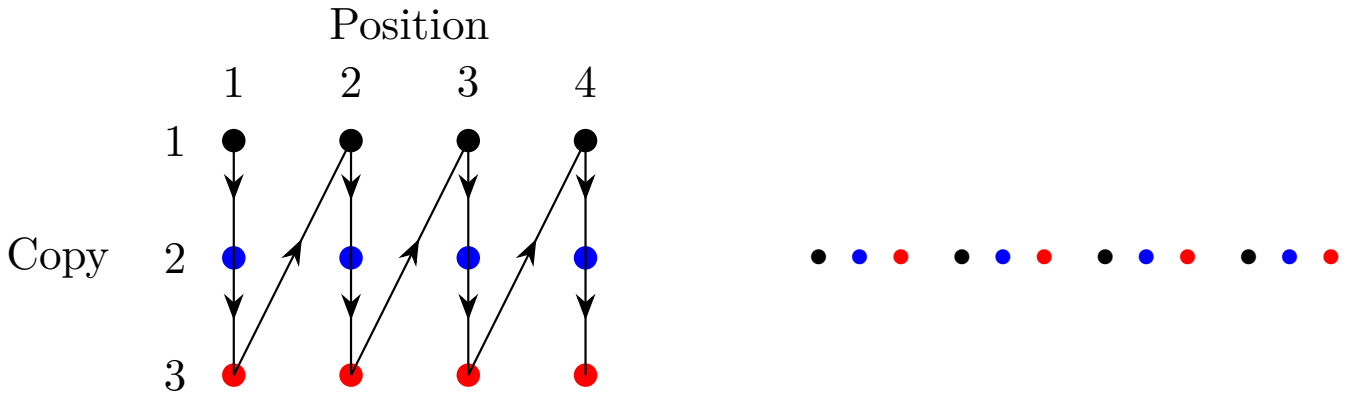
Uab Meto: /kokɪse/→ [kôîkse] 'the bread'

Input:

k o k ɪ s e

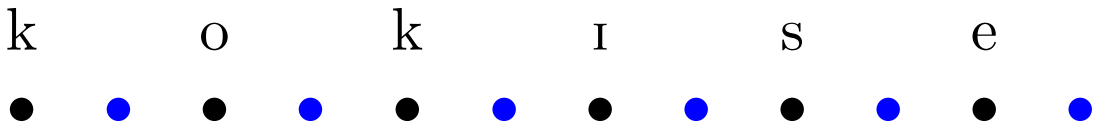
Copy and Delete, Position-wise

position-wise: order by position, then by copy

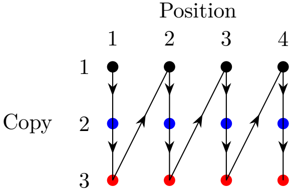


Uab Meto: /kokɪse/→ [kɔ͡ɔ͡ɪkse] 'the bread'

Output (make space):

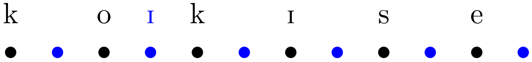


position-wise: order by position, then by copy



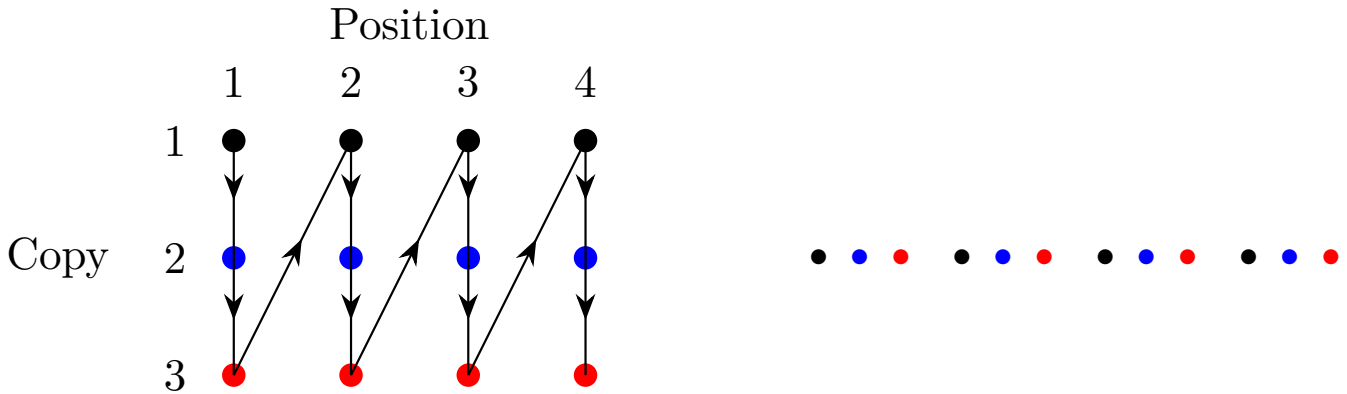
Uab Meto: /kokɪse/→ [ko͡ɪkse] ‘the bread’

Output (copy):



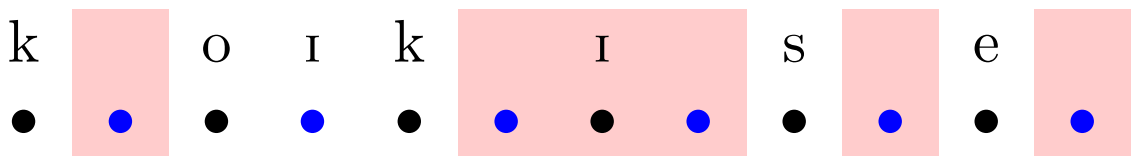
Copy and Delete, Position-wise

position-wise: order by position, then by copy



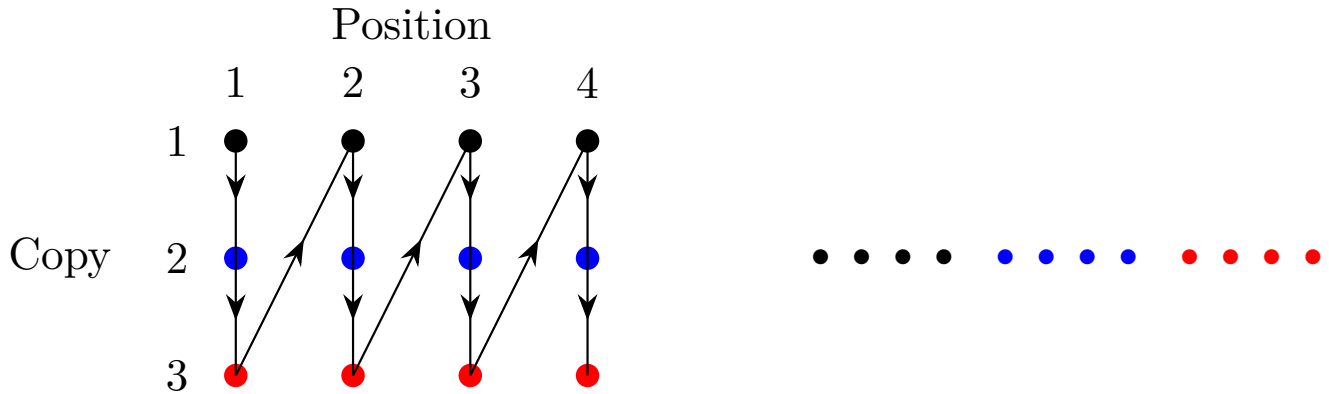
Uab Meto: /kokɪse/→ [kɔ̀ɔ̀ikse] ‘the bread’

Output (copy and delete):



Copy and Delete, Copy-wise

copy-wise: order by copy, then by position



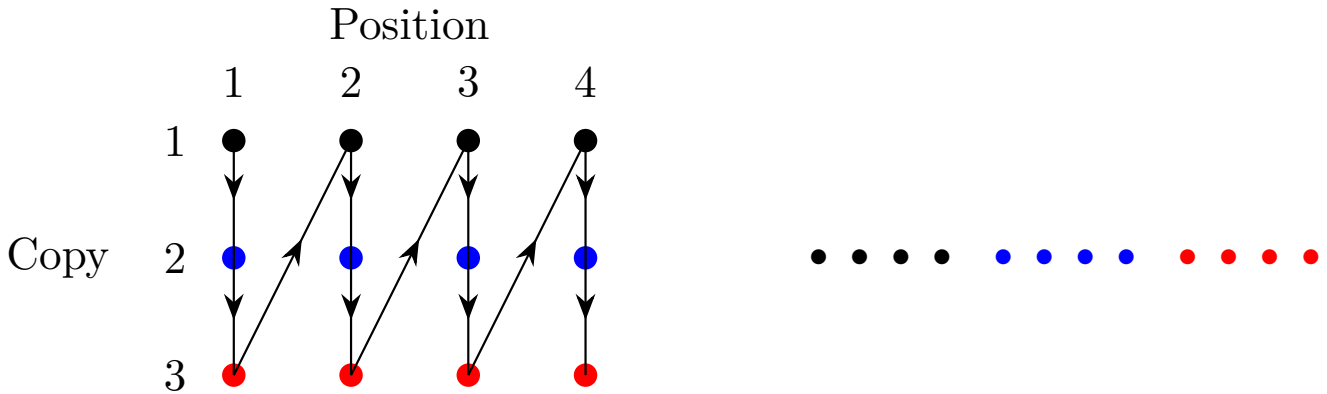
Uab Meto: /kokɪse/→ [kôĩkse] ‘the bread’

Input:

k o k ɪ s e

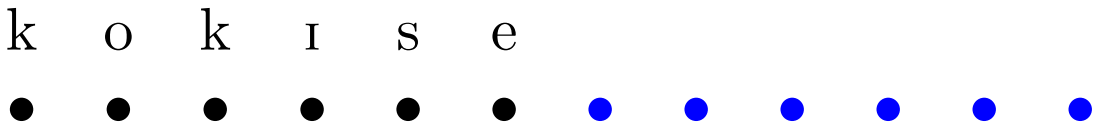
Copy and Delete, Copy-wise

copy-wise: order by copy, then by position



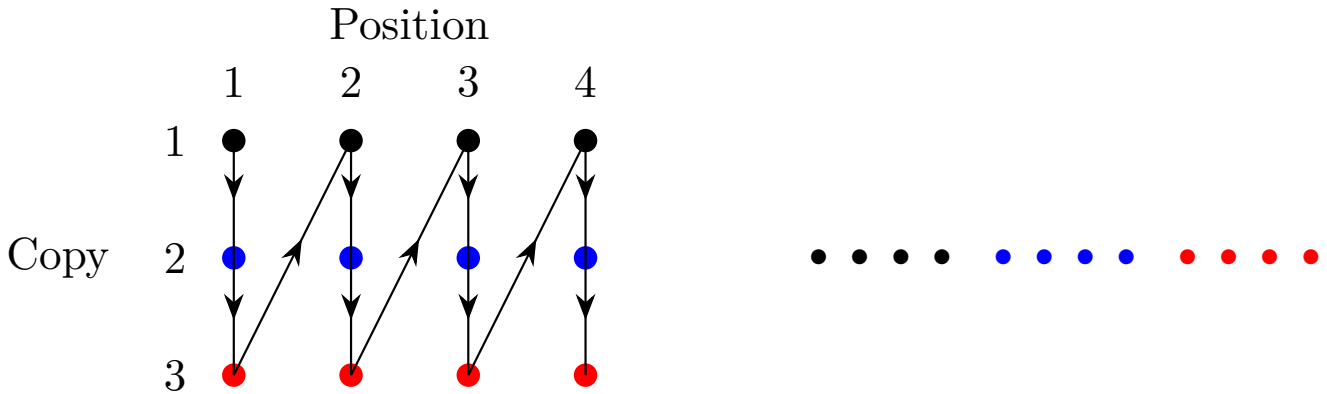
Uab Meto: /kokɪse/→ [kɔ̀ɔ̀ikse] 'the bread'

Output (make space):



Copy and Delete, Copy-wise

copy-wise: order by copy, then by position



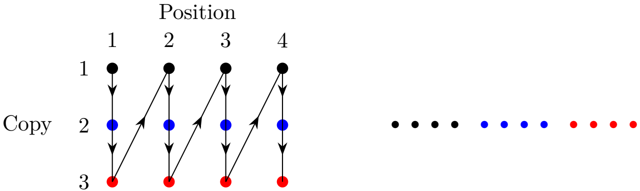
Uab Meto: /kokɪse/→ [kɔ̀ɔ̀ikse] ‘the bread’

Output (copy):

k	o	k	ɪ	s	e	k	o	k	ɪ	s	e
●	●	●	●	●	●	●	●	●	●	●	●

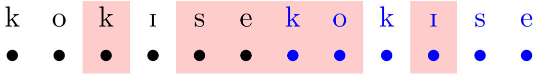
Copy and Delete, Copy-wise

copy-wise: order by copy, then by position



Uab Meto: /kokɪse/→ [ko͡ɪkse] 'the bread'

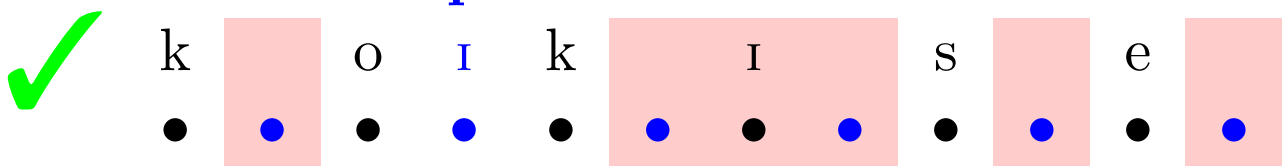
Output (delete):



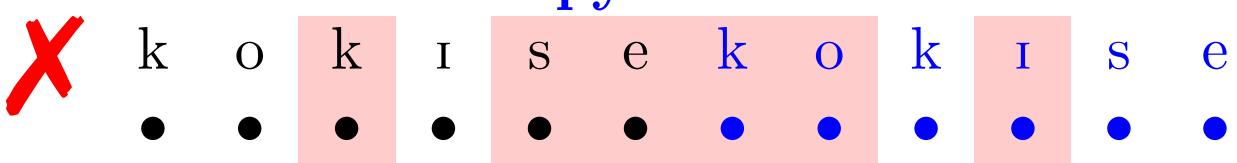
Observation #1

The position-wise ordered function resembles the copy-and-delete analyses of Takahasi (2019) and Mooney (2022), but the copy-wise ordering function does not.

position-wise



copy-wise

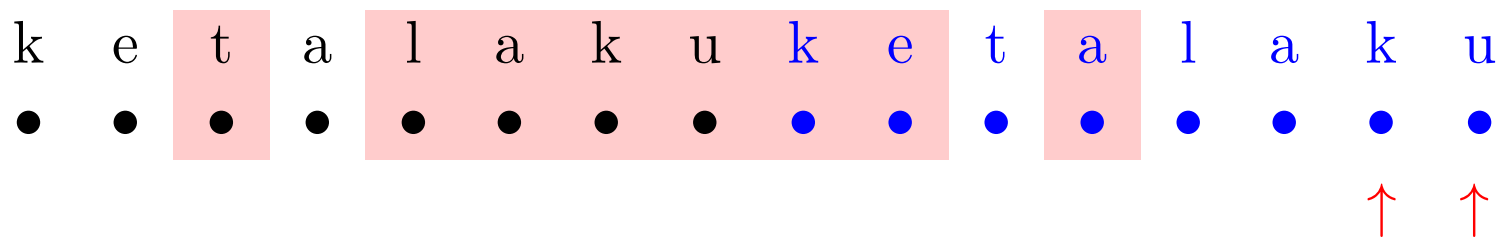


Observation #2

This copy-wise ordering necessarily *bounds* the number of instances of metathesis, regardless of the size of the input, because

1. the size of the copy is fixed in advance, and
2. copy-wise ordered functions requires a copy size $n + 1$ for n instances of metathesis.

Kwara'ae: /ketalaku/ → [k̟eatlau̟k] 'my height'

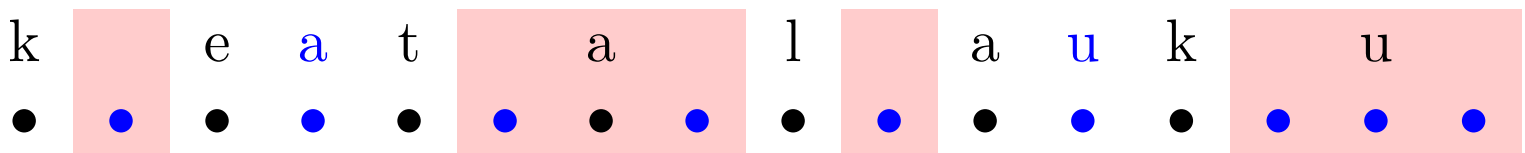


Need another copy to linearize [uk]!

Observation #3

In contrast, for position-wise ordered functions with copy size 2, it's the size of the input which bounds the number of instances of metathesis.

Kwara'ae: /ketalaku/ → [k͡eatla͡uk] 'my height'

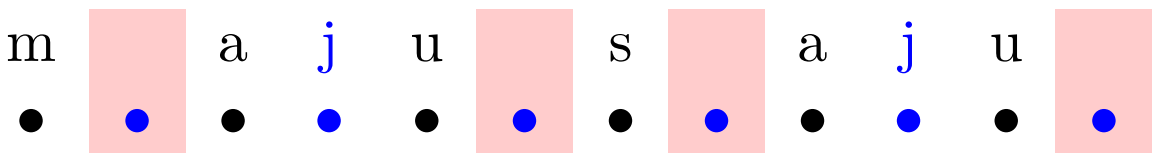


Observation #4

This difference between position-wise and copy-wise ordering is less about metathesis, and more about "making space for insertion."

Hypothetical: $\varnothing \longrightarrow$ j / a __ u

/mausau/ → [majusaju] 'logic'



Observation #5

More generally:

1. With position-wise orderings, it's easy to describe functions where the number of possible insertion sites scales with the input size. However, the inserted material itself is bounded in size. A copy set of size n allows a max $n-1$ positions to be inserted between input segments.

2. For copy-wise ordered functions, the number of possible insertion sites is fixed by the size of the copy set (so 2 for reduplication, 3 for triplication), but the size of the inserted material scales with the size of the input.

Observation #6

Caveat

- I have not proved that it is impossible to model multiple metathesis/epenthesis with copy-wise ordered functions.
- A proof will depend on the logical language. I conjecture it is impossible with FO(<). I am unsure about MSO(<).

Conclusion

1. Logical transductions in the style of Courcelle admit two kinds of orderings: position-wise and copy-wise.
2. While many processes can be modeled either way, certain phonological processes (epenthesis, metathesis) are more easily modeled position-wise, and certain morphological processes are more easily modeled copy-wise.
3. This supports the hypothesis that
   - **phonological** processes obey **position-wise** ordering.
   - **morphological** processes obey **copy-wise** ordering.
4. This analysis holds for other logical languages, such as Boolean Recursive Monadic Schemes (Chandlee and Jardine 2021).

Acknowledgments

I'd like to thank those who participated in the following at Stony Brook University:

- the Fall 2024 computational phonology seminar
- the Spring 2025 PhoRUM reading group
- the Spring 2025 Math Ling Recreation Group

Thank you too!

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