

# COMPUTATIONAL PHONOLOGY - CLASS 5

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- 3 Even FO logic *is not necessary* to describe phonological generalizations.
- 4 We argue weaker logics for constraints and transformations *are sufficient*.
- 5 We contrast this with the dominant paradigm in phonology: Optimality Theory.

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- ① Logic is not just another formalism.
- ② It usefully demarcates properties of extensions independent of *any* grammatical formalism.
- ③ Those properties are about what *any computer* (machine or organism) must be able to distinguish or not distinguish.
- ④ Logic classifies these properties along two dimensions: representation and computational power.

Part I

Regular?

# MSO-DEFINABLE CONSTRAINTS

## Some equivalences



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## Some equivalences

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$\equiv \text{1NFA} \equiv \text{1DFA} \equiv \text{2NFA} \equiv \text{2DFA}$

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# MSO-DEFINABLE CONSTRAINTS

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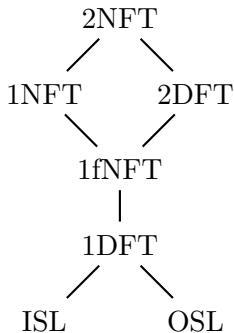
$$\equiv 1\text{NFA} \equiv 1\text{DFA} \equiv 2\text{NFA} \equiv 2\text{DFA}$$

$$\equiv \text{RE} \equiv \text{GRE}$$

**The extensions of these grammars are often called ‘regular’ or ‘rational’ formal languages/ constraints/ stringsets.**

(Kleene 1956, Scott and Rabin 1959, Büchi 1960, McNaughton and Papert 1971, Hopcroft and Ullman 1979, Thomas 1997)

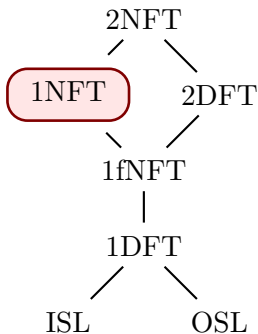
# MSO-DEFINABLE TRANSFORMATIONS



Properties separate when studying transformations.

(Engelfriedt and Hoogeboom 2001, Chandlee 2014, Chandlee et al. 2014, 2015, Filiot and Reynier 2016)

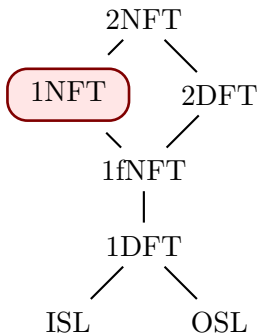
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Computational linguists call 1NFT **regular relations**.

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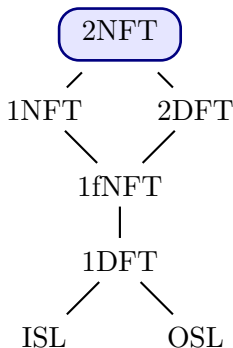
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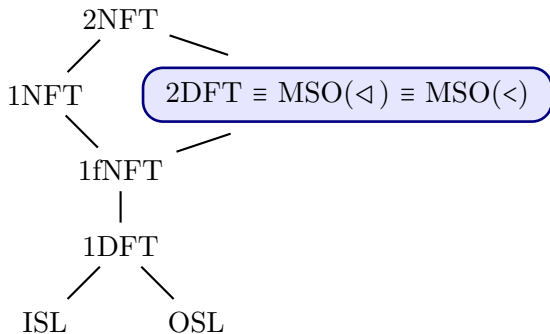


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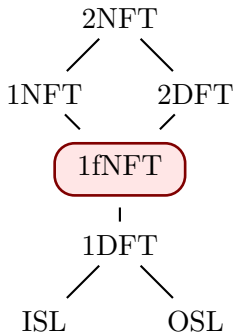
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French computer scientists call 2DFT **regular functions**.

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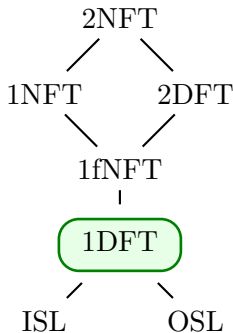
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French computer scientists call 1fNFT **rational functions**.

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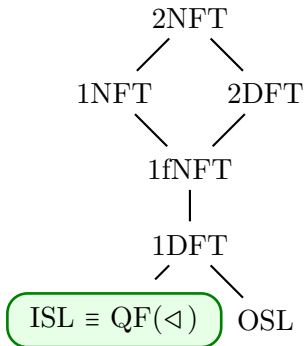
# MSO-DEFINABLE TRANSFORMATIONS



French computer scientists call 1DFT **sequential functions**.

(Engelfriedt and Hoogeboom 2001, Chandlee 2014, Chandlee et al. 2014, 2015, Filiot and Reynier 2016)

# MSO-DEFINABLE TRANSFORMATIONS

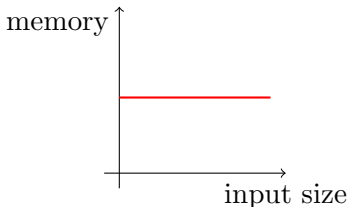


Today we are going to talk a lot about this class of functions!

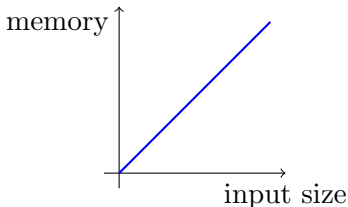
(Engelfriedt and Hoogeboom 2001, Chandlee 2014, Chandlee et al. 2014, 2015, Filiot and Reynier 2016)

## WHAT “REGULAR” MEANS

A set, relation, or function is **regular** provided **the memory required for the computation is bounded by a constant, regardless of the size of the input.**



Regular



Non-regular

# EXAMPLE: VOWEL HARMONY

## Progressive

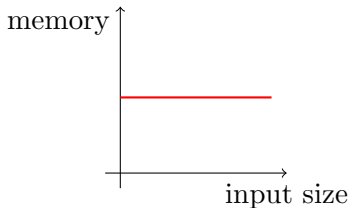
*Vowels agree in backness with the first vowel in the underlying representation.*

## Majority Rules

*Vowels agree in backness with the majority of vowels in the underlying representation.*

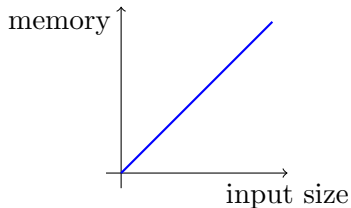
UR	Progressive	Majority Rules
/nokelu/	nok <u>o</u> lu	nok <u>o</u> lu
/nokeli/	nok <u>o</u> lu	nik <u>e</u> li
/pidugo/	pid <u>i</u> ge	pu <u>u</u> dugo
/pidugomemi/	pid <u>i</u> gememi	pid <u>i</u> gememi

# PROGRESSIVE AND MAJORITY RULES HARMONY



Regular

Progressive



Non-regular

Majority Rules

# SOME PERSPECTIVE

TYPOLOGICAL: Majority Rules is unattested.

(Lombardi 1999, Bakovic 2000, Bowler 2013)

PSYCHOLOGICAL: Human subjects fail to learn Majority Rules  
in artificial grammar learning experiments, unlike  
progressive harmony. (Finley 2008, 2011)

COMPUTATIONAL: Majority Rules is not regular.

(Riggle 2004, Heinz and Lai 2013)



# OPTIMALITY THEORY

- 1 There exists a CON and ranking over it which generates Majority Rules:  $\text{AGREE}(\text{BACK}) \gg \text{IDENTIO}[\text{BACK}]$ .
- 2 Changing CON may resolve this, but this solution misses the forest for the trees.

## Part II

# Phonological Theories

# DESIDERATA FOR PHONOLOGICAL THEORIES

- 1 Provide a theory of typology
  - Be sufficiently expressive to capture the range of cross-linguistic phenomenon  
(explain what is there)
  - Be restrictive in order to be scientifically sound  
(explain what is not there)
- 2 Provide learnability results  
(explain how what is there could be learned)
- 3 Provide insights  
(for example: grammars should distinguish marked structures from their repairs)
- 4 Effectively computable

# MAIN CLAIM

- Particular *sub-regular* computational properties—and not optimization—**best** characterize the nature of phonological generalizations.

## Part III

# Phonological Generalizations are Regular

# PHONOLOGICAL GENERALIZATIONS ARE REGULAR

Evidence supporting the hypothesis that phonological generalizations are finite-state originate with Johnson (1972) and Kaplan and Kay (1994), who showed how to translate any phonological grammar defined by an ordered sequence of SPE-style rewrite rules into a 1NFT (rational relation).

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## Consequently:

- 1 Constraints on well-formed surface and underlying representations are regular (since the image and pre-image of rational relations are regular). (Rabin and Scott 1959)



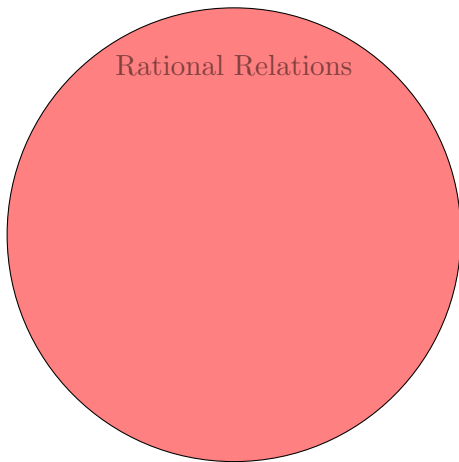
# PHONOLOGICAL GENERALIZATIONS ARE REGULAR

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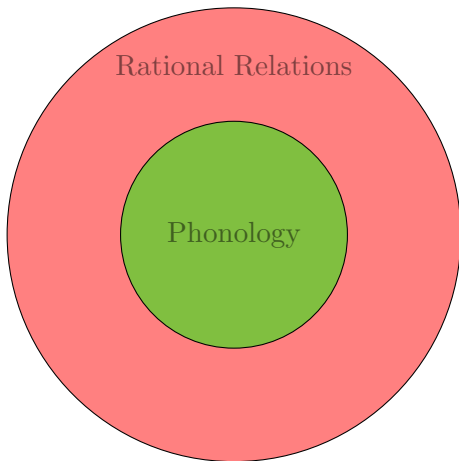
## Consequently:

- 1 Constraints on well-formed surface and underlying representations are regular (since the image and pre-image of rational relations are regular). (Rabin and Scott 1959)
- 2 Since virtually any phonological grammar can be expressed as an ordered sequence of SPE-style rewrite rules, this means “being regular” is a property of the function that *any* phonological grammar defines.

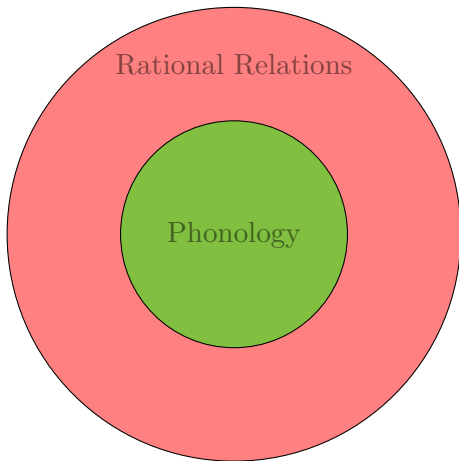
# IN PICTURES



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Some argued rule-based grammars overgenerated. Also, nobody knew how to learn them.

# Part IV

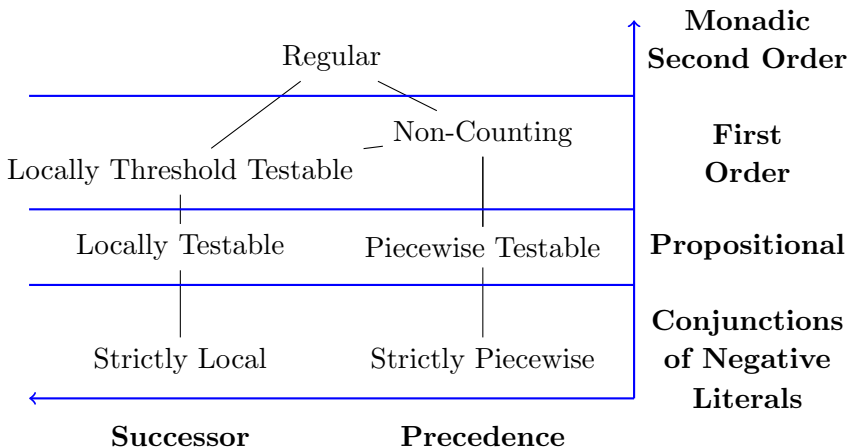
## Analytical Framework

# COMPUTATION IS REFLECTED IN LOGICAL POWER

*Subregular* hierarchies organize pattern complexity along two dimensions.

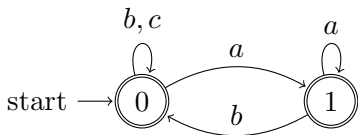
- **logical power** along the vertical axis
- **representational primitives** along the horizontal axis.

# LOGICAL CHARACTERIZATIONS OF SUBREGULAR STRINGSETS

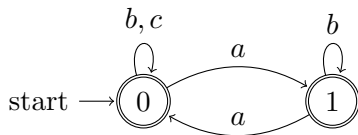


(McNaughton and Papert 1971, Rogers and Pullum 2011, Rogers et al. 2013, Heinz 2018)

# SIZE OF AUTOMATA $\propto$ COMPLEXITY? NO.



**G1**



**G2**

- G1 maintains a short term memory w.r.t. [a] (i.e. State 1 means “just observed [a]”).
- G2 maintains a memory of the even/odd parity of [a]s (i.e. State 1 means “observed an even number of [a]s”).
- G1 generates/recognizes all words except those with a forbidden string [ac]; and G2 generates/recognizes all words except those with a [c] whose left context contains an even number of [a]s. G1 is Strictly 2-Local, and G2 is properly regular.

(Heinz and Idsardi 2013)



# LOGIC AS A HIGH-LEVEL LANGUAGE

- ① Logical formulas over relational structures (model theory) provide a high-level description language (which are easy to learn to write—even for whole grammars).
- ② We argue these levels of complexity yield hypotheses characterizing phonology that provide
  - ① a better fit to the typology than optimization,
  - ② have learning results that are as good or better than in OT,
  - ③ provide equally good or better insights,
  - ④ and are effectively computable.

## Part V

# Input Strictly Local Functions

# INPUT STRICT LOCAL *Transformations*

This is a class of transformations which...

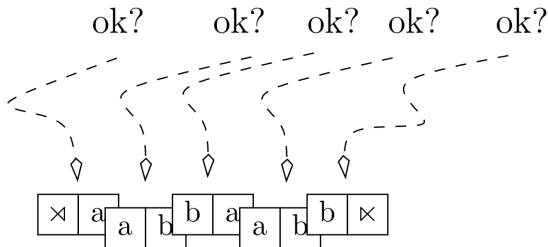
- ① generalizes Strictly Local Stringsets,
- ② captures a wide range of phonological phenomena,
- ③ including *opaque* transformations,
- ④ and is effectively learnable!

(Chandlee 2014, Chandlee et al. 2014, 2015, Chandlee and Heinz 2018, Chandlee et al. 2018)

# STRICTLY LOCAL CONSTRAINTS FOR STRINGS

When words are represented as strings, **local sub-structures are sub-strings** of a certain size.

Here is the string *abab*. If we fix a diameter of 2, we have to check these substrings.



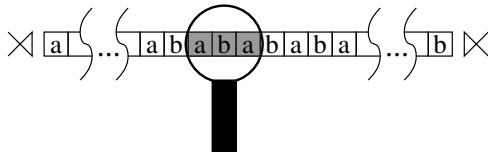
An ill-formed sub-structure is **forbidden**.

(Rogers and Pullum 2011, Rogers et al. 2013)

# STRICTLY LOCAL CONSTRAINTS FOR STRINGS

When words are represented as strings, **local sub-structures are sub-strings** of a certain size.

- We can imagine examining each of the local-substructures, checking to see if it is forbidden or not. The whole structure is well-formed only if each local sub-structure is.

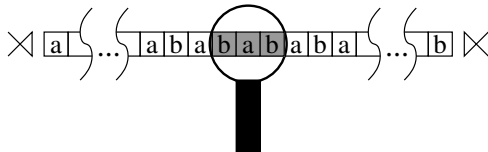


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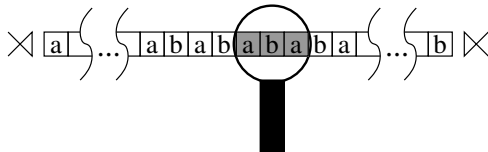


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(Rogers and Pullum 2011, Rogers et al. 2013)

# EXAMPLES

## Examples of Strictly Local constraints for strings

- \*aa
- \*ab
- \*NC<sub>0</sub>
- NoCoDA

## Examples of Non-Strictly Local constraints

- \*s...f (Hansson 2001, Rose and Walker 2004, Hansson 2010)
- \*#s...f# (Lai 2012, 2015)
- Obligatoriness: Words must contain one primary stress (Hayes 1995, Hyman 2011, inter alia).



# SL CONSTRAINTS AS CONJUNCTIONS OF NEGATIVE LITERALS

$$\varphi \stackrel{\text{def}}{=} \neg np \wedge \neg nk \wedge \neg mt \wedge \neg mg$$

- A  $SL_k$  grammar can be thought of as a list of **forbidden** sub-strings whose max length is  $k$ .
- As a logical language, this is simply the conjunctions of negative literals:
  - where the literals  $u$  are strings
  - and  $\mathcal{M}_w^{\triangleleft} \models u$  iff  $\mathcal{M}_u^{\triangleleft}$  is a sub-structure of  $\mathcal{M}_w^{\triangleleft}$ .

# ABSTRACT CHARACTERIZATIONS. . .

- 1 are independent of logical formulas, grammars, and automata
- 2 provide laws of *inference* for learning
- 3 provide ways to show certain stringsets do NOT belong to the class.

(Rogers and Pullum 2011, Rogers et al. 2013)

# SL STRINGSETS - ABSTRACT CHARACTERIZATION

The theorem below establishes a set-based characterization of SL stringsets independent of any grammar, scanner, or automaton.

## Theorem. $k$ -Local Suffix Substitution Closure

For all  $L \subseteq \Sigma^*$ ,  $L \in \text{SL}$  iff there exists  $k$  such that for all  $u_1, v_1, u_2, v_2, x \in \Sigma^*$  it is the case that

if  $u_1xv_1, u_2xv_2 \in L$  and  $|x| = k - 1$   
then  $u_1xv_2 \in L$

# USING SUFFIX SUBSTITUTION CLOSURE

- The theorem provides a law which simultaneously
  - provides a basis for *inference*
  - provides a method for establishing non- $SL_k$  stringsets.

$$\frac{\begin{array}{c|c|c} u_1 & \sigma_1 \cdots \sigma_{k-1} & v_1 \in L \\ u_2 & \sigma_1 \cdots \sigma_{k-1} & v_2 \in L \end{array}}{u_1 \mid \sigma_1 \cdots \sigma_{k-1} \mid v_2 \in L}$$

# IN-CLASS EXERCISES

- 1 Consider a Strictly 2-Local stringset  $L$  which contains the words  $aa$  and  $ab$ . Using Suffix Substitution Closure, explain what other words must be in  $L$ .
- 2 Consider the constraint  $*s\dots j$ . Show this is not  $SL_k$  for any  $k$ .

# COGNITIVE INTERPRETATION OF SL

- Any cognitive mechanism that can distinguish member strings from non-members of a  $SL_k$  stringset must be sensitive, at least, to the length  $k$  blocks of consecutive events that occur in the presentation of the string.
- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to the immediately prior sequence of  $k - 1$  events.
- Any cognitive mechanism that is not sensitive to the length  $k$  blocks of consecutive events that occur in the presentation of the string will be unable to recognize some  $SL_k$  stringsets.

(Rogers et al. 2013)

# INPUT STRICT LOCAL *Transformations*

This is a class of transformations which...

- ① generalizes Strictly Local Stringsets,
- ② captures a wide range of phonological phenomena,
- ③ including *opaque* transformations,
- ④ and is effectively learnable!

(Chandlee 2014, Chandlee et al. 2014, 2015, Chandlee and Heinz 2018)

# INPUT STRICT LOCALITY: MAIN IDEA

These transformations are Markovian in nature.

$$\begin{array}{cccc} x_0 & x_1 & \dots & x_n \\ & & \downarrow & \\ u_0 & u_1 & \dots & u_n \end{array}$$

where

- 1 Each  $x_i$  is a single symbol  $(x_i \in \Sigma_1)$
- 2 Each  $u_i$  is a *string*  $(u_i \in \Sigma_2^*)$
- 3 There exists a  $k \in \mathbb{N}$  such that for all input symbols  $x_i$  its output string  $u_i$  depends only on  $x_i$  and the  $k - 1$  elements immediately preceding  $x_i$ .  
(so  $u_i$  is a function of  $x_{i-k+1}x_{i-k+2} \dots x_i$ )



# INPUT STRICT LOCALITY: MAIN IDEA IN A PICTURE

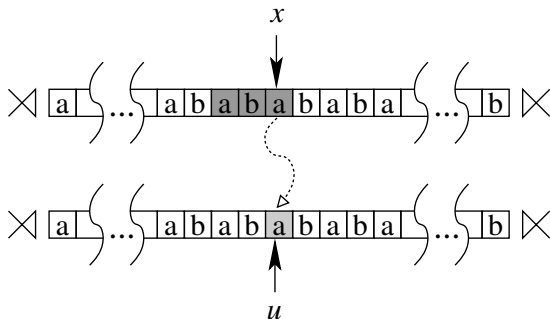


FIGURE: For every Input Strictly  $k$ -Local function, the output string  $u$  of each input element  $x$  depends only on  $x$  and the  $k - 1$  input elements previous to  $x$ . In other words, the contents of the lightly shaded cell only depends on the contents of the darkly shaded cells.

EXAMPLE: WORD-FINAL /E/ RAISING IS ISL WITH  
 $k = 2$

/ove/  $\mapsto$  [ovi]

input:	⊗	o	v	e	⊗
output:	⊗	o	v	λ	i ⊗

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input:	⊗		o		v		e		⊗
<hr/>									
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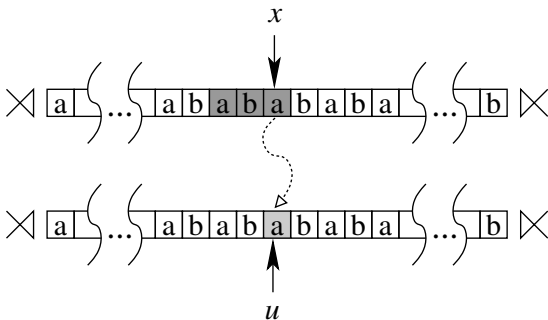
/ove/  $\mapsto$  [ovi]

input:	⊗	o	v	e	⊗
output:	⊗	o	v	λ	i ⊗

## WHAT THIS MEANS, GENERALLY.

The *necessary information* to decide the output is contained within a *window of bounded length* on the *input* side.

- This property is largely independent of whether we describe the transformation with constraint-based grammars, optimization-based grammars, rule-based grammars, or other kinds of grammars.



# HOW DOES THIS RELATE TO TRADITIONAL PHONOLOGICAL GRAMMATICAL CONCEPTS?

- 1 Like OT,  $k$ -ISL functions do not make use of intermediate representations.
- 2 Like OT,  $k$ -ISL functions separate marked structures from their repairs (Chandlee et al. 2014, AMP).
  - $k$ -ISL functions are sensitive to all and only those markedness constraints which could be expressed as  $*x_1x_2\dots x_k$ , ( $x_i \in \Sigma$ ).  
(So Strictly  $k$ -Local markedness constraints)
  - In this way,  $k$ -ISL functions model the “homogeneity of target, heterogeneity of process” (McCarthy 2002)

# Part VI

## Learning ISL functions



# RESULTS IN A NUTSHELL

- Particular finite-state transducers can be used to represent ISL functions.
- Grammatical inference techniques (de la Higuera 2010) are used for learning.
- **Theorems:** Given  $k$  and a sufficient sample of  $(u, s)$  pairs any  $k$ -ISL function can be *exactly* learned in *polynomial* time and data.
  - ISLFLA (Chandlee et al. 2014, TACL) (quadratic time and data)
  - SOSFIA (Jardine et al. 2014, ICGI) (linear time and data)

# COMPARISON OF LEARNING RESULTS IN CLASSIC OT

- Recursive Constraint Demotion (RCD) is guaranteed to give you a consistent grammar in reasonable time.
- Exact convergence is not guaranteed for RCD because the nature of the data sample needed for exact convergence is not yet known.
- On the other hand, we are able to characterize a sample which yields exact convergence.

## Part VII

# ISL Functions and Phonological Typology

# WHAT CAN BE MODELED WITH ISL FUNCTIONS?

1. Many individual phonological processes.  
(local substitution, deletion, epenthesis, and metathesis)

**Theorem:** Transformations describable with a rewrite rule

R:  $A \rightarrow B / C \_ D$  where

- CAD is a finite set,
- R applies simultaneously, and
- contexts, but not targets, can overlap

are ISL for  $k$  equal to the longest string in CAD.

(Chandlee 2014, Chandlee and Heinz, 2018)

# WHAT CAN BE MODELED WITH ISL FUNCTIONS?

2. Approximately 95% of the individual processes in P-Base (v.1.95, Mielke (2008))
3. Many *opaque* transformations *without* any special modification.

(Chandlee 2014, Chandlee and Heinz 2018)

# OPAQUE ISL TRANSFORMATIONS

- Opaque maps are typically defined as the extensions of particular rule-based grammars (Kiparsky 1971, McCarthy 2007). Tesar (2014) defines them as *non-output-driven*.
- Baković (2007) provides a typology of opaque maps.
  - Counterbleeding
  - Counterfeeding on environment
  - Counterfeeding on focus
  - Self-destructive feeding
  - Non-gratuitous feeding
  - Cross-derivational feeding
- Each of the examples in Baković's paper is ISL.

(Chandlee et al. 2018)

# EXAMPLE: COUNTERBLEEDING IN YOKUTS

	‘might fan’
	<hr/>
	/ʔili:+1/
	<hr/>
[+long] → [-high]	ʔile:l
V → [-long] / — C#	ʔilel
	<hr/>
	[ʔilel]

# EXAMPLE: COUNTERBLEEDING IN YOKUTS IS ISL WITH $k=3$

$/\text{?ili:l}/ \mapsto [\text{?ilel}]$

input:	×	?	i	l	i:	l	×
output:	×	?	i	l	λ	λ	el ×



# EXAMPLE: COUNTERBLEEDING IN YOKUTS IS ISL WITH $k=3$

$/ʔi:lil/ \mapsto [ʔilel]$

input:	ɤ	ʔ	i	l	i:	l	ɤ
output:	ɤ	ʔ	i	l	λ	λ	el ɤ

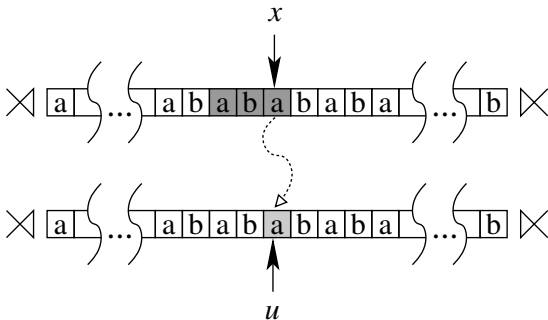
# EXAMPLE: COUNTERBLEEDING IN YOKUTS IS ISL WITH $k=3$

$/\text{?ili:l}/ \mapsto [\text{?ilel}]$

input:	$\times$	?	i	l	i:	l	$\times$
output:	$\times$	?	i	l	$\lambda$	$\lambda$	el $\times$

## INTERIM SUMMARY

Many phonological patterns, including many opaque ones, have the *necessary information* to decide the output contained within a *window of bounded length* on the *input side*.



And can thus be learned by the ISLFLA and SOSFIA algorithms!

# LOGICAL CHARACTERIZATION OF ISL FUNCTIONS

## Theorem.

ISL functions correspond exactly to the functions definable with Quantifier-Free logic with successor:  $\text{QF}(\triangleleft)$ .

(Lindell and Chandler, LICS 2016)

# LOGICAL CHARACTERIZATION OF ISL FUNCTIONS

## Theorem.

ISL functions correspond exactly to the functions definable with Quantifier-Free logic with successor:  $\text{QF}(\triangleleft)$ .

Quantifier-free formulas are ones without *any* quantification!

(Lindell and Chandler, LICS 2016)

# LOCALITY AND QUANTIFICATION

Compare:

①  $P(x) \stackrel{\text{def}}{=} Q(x) \wedge \exists y[R(y)]$  (First Order Definable)

Requires scanning whole word for such a  $y$ !!

②  $P(x) \stackrel{\text{def}}{=} Q(x) \wedge R(\text{predecessor}(x))$  (QF Definable)

Information to decide  $P$  is local to  $x$  in the input!!

# AUTOSEGMENTAL INPUT STRICTLY LOCAL FUNCTIONS

- Defined as Quantifier Free Transformations with Autosegmental Representations (ASR)
- To what extent does ASRs make non-local processes local?

Pattern	Language	ISL	A-ISL
Bounded shift (§4.1, 5.2)	Rimi	✓	✓
Bounded spread (§6.1)	Bemba	✓	✓
Bounded Meussen's Rule (§6.3)	Luganda	✓	✗
Unbounded shift (§4.2,5.3)	Zigula	✗	✓
Unbounded deletion (§6.2)	Arusa	✗	✓
Alternating Meussen's Rule (§6.4)	Shona	✗	✗
Unbounded spread (§6.5)	Ndebele	✗	✗

Table 1: Summary of analyses.

(Chandlee and Jardine 2019, TACL)

# WHAT CANNOT BE MODELED WITH ISL FUNCTIONS

- ① progressive and regressive spreading
- ② long-distance (unbounded) consonant and vowel harmony
- ③ non-*regular* transformations like Majority Rules vowel harmony and non-*subsequential* transformations like Sour Grapes vowel harmony (Baković 2000, Finley 2008, Heinz and Lai 2013)

(Chandlee 2014, Chandlee and Heinz 2018)



# TYPOLOGICAL SUMMARY OF ISL

## Undergeneration? Yes, for now...

- ISL functions are insufficiently expressive for spreading and long-distance harmony. I will discuss these later (or in Q&A).

## Overgeneration? Not so much.

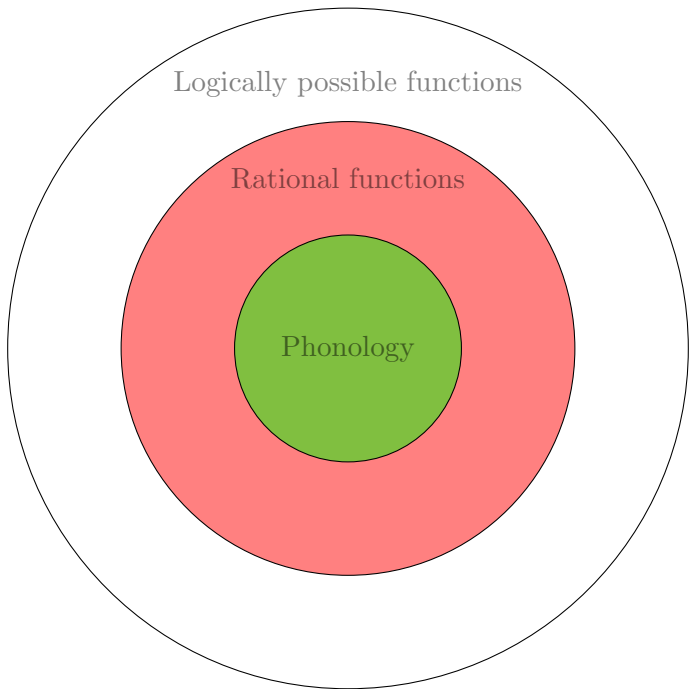
- **Theorem:** ISL is a proper subclass of left and right subsequential functions.  
(Chandlee 2014, Chandlee et al. 2014)
- **Corollary:** SG and MR are not ISL for any  $k$ .  
(Heinz and Lai 2013)
- So MR and SG are correctly predicted to be outside the typology.

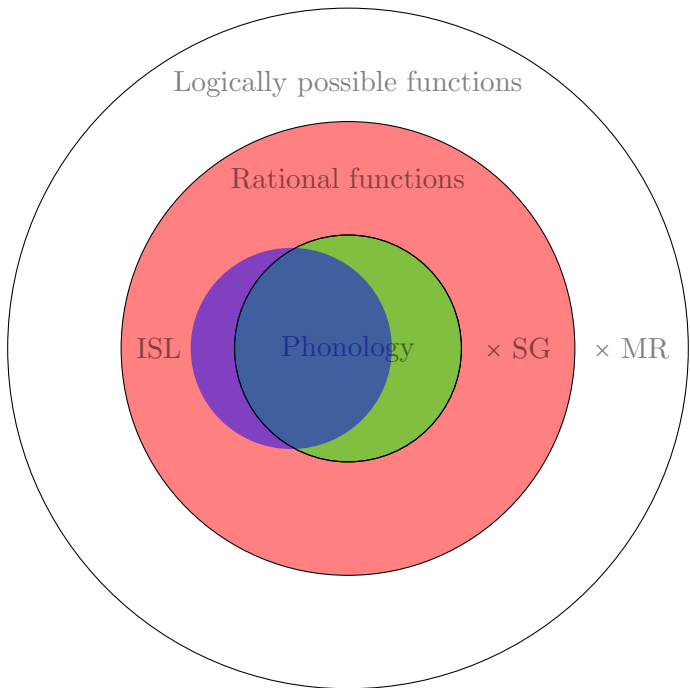


Logically possible functions

A Venn diagram consisting of two concentric circles. The outer circle is white with a black outline and is labeled "Logically possible functions". The inner circle is filled with a solid red color and is labeled "Rational functions". The inner circle is entirely contained within the outer circle, illustrating that rational functions are a subset of logically possible functions.

Rational functions





# UNDERGENERATION IN CLASSIC OT

- It is well-known that classic OT cannot generate opaque maps (Idsardi 1998, 2000, McCarthy 2007, Buccola 2013) (though Baković 2007, 2011 argues for a more nuanced view).
- Many, many adjustments to classic OT have been proposed.
  - constraint conjunction (Smolensky), sympathy theory (McCarthy), turbidity theory (Goldrick), output-to-output representations (Benua), stratal OT (Kiparsky, Bermudez-Otero), candidate chains (McCarthy), harmonic serialism (McCarthy), targeted constraints (Wilson), contrast preservation (Łubowicz) comparative markedness (McCarthy) serial markedness reduction (Jarosz), ...

See McCarthy 2007, *Hidden Generalizations* for review, meta-analysis, and more references to these earlier attempts.

# ADJUSTMENTS TO CLASSIC OT

The aforementioned approaches invoke different representational schemes, constraint types and/or architectural changes to classic OT.

- The typological and learnability ramifications of these changes is not yet well-understood in many cases.
- On the other hand, *no special modifications are needed* to establish the ISL nature of the opaque maps we have studied.

# OVERGENERATION IN CLASSIC OT

- It is not controversial that classic OT generates non-regular maps with simple constraints (Frank and Satta 1998, Riggle 2004, Gerdemann and Hulden 2012, Heinz and Lai 2013) (Majority Rules vowel harmony is one example.)

# SIMPLE CONSTRAINTS IN OT GENERATE NON-REGULAR MAPS

IDENT, DEP  $\gg$  \*ab  $\gg$  MAX

$$a^n b^m \mapsto a^n, \text{ if } m < n$$

$$a^n b^m \mapsto b^m, \text{ if } n < m$$

(Gerdemann and Hulden 2012)



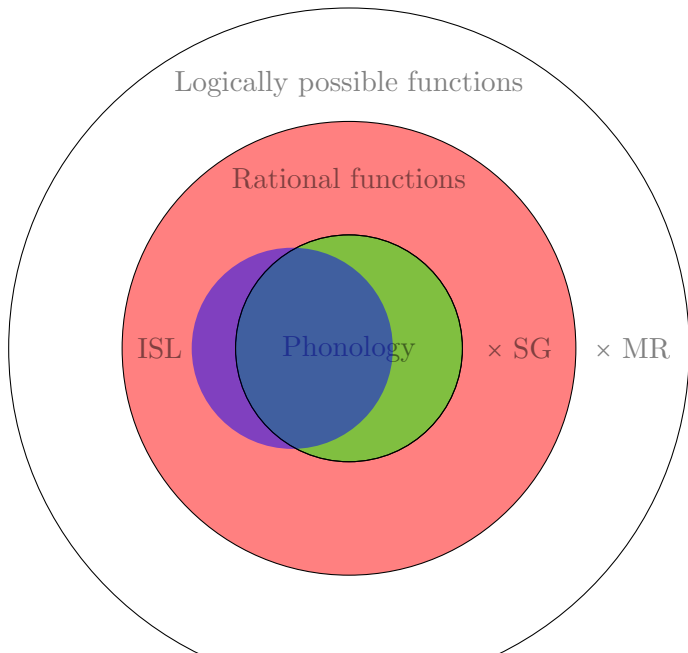
# OPTIMIZATION MISSES AN IMPORTANT GENERALIZATION

- When computing the output of phonological transformations, the *necessary information* is contained within *sub-structures of bounded size*.
- This is neither expected nor predicted under global optimization.
- On the other hand, it is one of the defining characteristics of *k*-ISL.

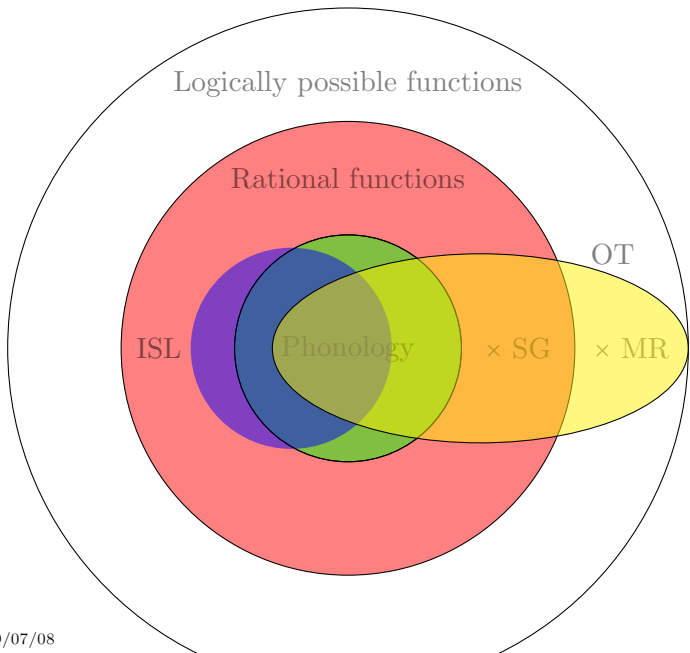
# OT'S GREATEST STRENGTH IS ITS GREATEST WEAKNESS.

- The signature success of a successful OT analysis is when complex phenomena are understood as the interaction of simple constraints.
- But the overgeneration problem is precisely this problem: complex—but weird—phenomena resulting from the interaction of simple constraints (e.g. Hansson 2007, Hansson and McMullin 2014, on ABC).
- As for the undergeneration problem, opaque candidates are not optimal in classic OT.

# CLASSIC OT AND TYPOLOGY



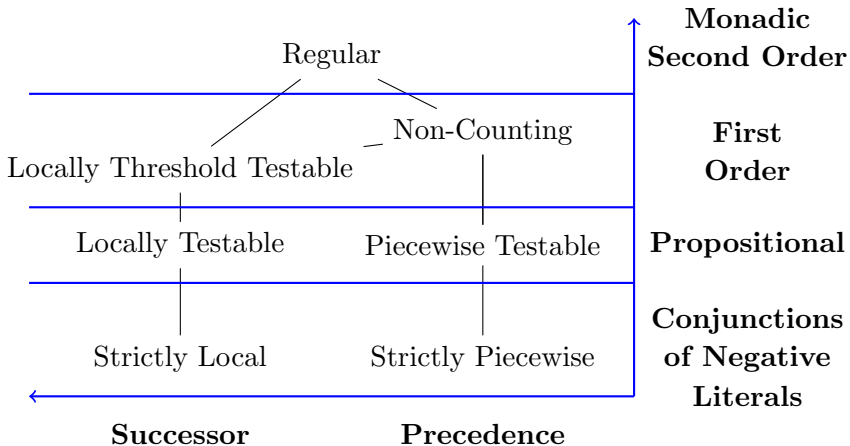
# CLASSIC OT AND TYPOLOGY



# Part VIII

## Conclusion

# LOGICAL CHARACTERIZATIONS OF SUBREGULAR STRINGSETS



(McNaughton and Papert 1971, Rogers and Pullum 2011, Rogers et al. 2013)

## SOME CONCLUSIONS

- $k$ -ISL functions provide both a more expressive and restrictive theory of typology than classic OT, which we argue **better matches** the attested typology.
  - In particular: Many phonological transformations, **including opaque ones**, can be expressed with them, but non-subsequential transformations cannot be.
- $k$ -ISL functions are feasibly learnable.
- Like classic OT, there are no intermediate representations, and  $k$ -ISL functions can express the “homogeneity of target, heterogeneity of process” which helps address the conspiracy and duplication problems.
- Unlike OT, *subregular computational properties* like ISL—and not optimization—form the core computational nature of phonology.

# HOMEWORK FOR THURSDAY

- 1 Read Strother-Garcia (2018) “Imdlawn Tashlhiyt Berber Syllabification is Quantifier-Free” (8 pages)
- 2 Read Chandlee and Jardine (2019) “Autosegmental Input Strictly Local Functions” (12 pages)

## Supplemental reading

- Chandlee and Heinz (2018) “Strict Locality and Phonological Maps”
- Rawski (under review) “The Logical Nature of Phonology Across Speech and Sign”